FINITE ELEMENT ANALYSIS OF ELASTO PLASTIC DYNAMIC BEHAVIOUR OF PLATES UNDER IMPACT LOADING

by

Praveen Pandey

DEPARTMENT OF MECHANICAL ENGINEERING

INDIAN INSTITUTE OF TECHNOLOGY KANPUR

FEBRUARY, 1996

FINITE ELEMENT ANALYSIS OF ELASTO PLASTIC DYNAMIC BEHAVIOUR OF PLATES UNDER IMPACT LOADING

A Thesis Submitted
in Partial Fulfillment of the requirement
For the Degree of
MASTER OF TECHNOLOGY

by

PRAVEEN PANDEY

to the

DEPARTMENT OF MECHANICAL ENGINEERING
INDIAN INSTITUTE OF TECHNOLOGY, KANPUR

FEBRUARY, 1996

CERTIFICATE

12-2-96

This is to certify that the present work titled ' **Finite Element Analysis Of Elasto Plastic Dynamic Behaviour Of Plates Under Impact Loading** ' by **Praveen Pandey** has been carried out under my supervision and has not been submitted elsewhere for the award of degree.

Dr. N. N. KISHORE
Professor
Mechanical Engg. Dept.
I. I. T. Kanpur
INDIA

February, 1996

ACKNOWLEDGEMENTS

I express my deep sense of gratitude and indebtedness to Prof. N. N. Kishore for his invaluable guidance and constructive suggestions throughtout the present work.

I am grateful to my friends Rajesh Patnaik, S. K. Verma and Sanjeev Deshmukh for their help in one or other way.

It gives me immense pleasure to recall my association with Gagan, Raj, Sukku, Amit, Ramayan, Manoranjan, Sudipto, Madhukar, Amarendra and all other friends who made my stay at IITK a pleasant memory.

I wish to thank Prof. P. Kumar and his M.Tech. students for their co-operation in providing lab facilities at peak times.

And finally I pay my respects to All Mighty who provided me enough inner strength to carry on my work.

PRAVEEN PANDEY

February, 1996

Dedicated to

MY PARENTS

CONTENTS

NOTATIONS

A	Part of elemental stiffness matrix
AA	Slope of stress strain curve
{a}	Flow vector
a0-a7	Constant used in Newmark's Implicit scheme
B	Part of elemental stiffness matrix
BB	Matrix relating strains and displacements
C	A part of elemental stiffness matrix
C_1	Property matrix
c	Dilatational wave velocity
c2	Shear wave velocity
D	A part of elemental stiffness matrix
DD	Stress strain matrix
DD^{ep}	Elastoplastic stress strain matrix
E	A part of stress strain matrix
E	Modulous of elasticity
[F]	A part of elemental mass matrix
{f}	Load vector
G	Shear modulous of elasticity
h	Thickness
h_p	Elastic thickness
[K]	Stiffness matrix
$[K]^{ep}$	Elastoplastic stiffness matrix
L	Differential operator
L_w	Critical wave length
L_e	Effective length of an element
[M]	Mass matrix
{M}	Moment vector
N	Shape function

P	A part of elemental stiffness matrix
q	Uniformly distributed load
S	Shear force
T	Edge moment
$\{U\}$	Displacement vector
u	Displacement in X-direction
v	Displacement in Y-direction
w	Displacement in Z-direction
δ,α	Constants used in Newmark's Implicit scheme
ϵ	Strain
Γ	Boundary
γ	Shear strain
ν	Poisson's ratio
σ	Stress
θ	Rotation
Ω	Area
υ	Differential operator
Δt	Time step

ABSTRACT

In practical cases when a plate is subjected to a dynamic loading , the geometric and material linearity may not be preserved. This study is aimed to investigate the effect of plasticity in the dynamic response . As there are no closed form solutions available owing to the complexity of the problem ,numerical methods are used to solve the problem.

For this computational work an efficient FEM code is developed using the "robust triangular element" developed by Zienkiewiz [1991] . "Mindlin plate theory" in the form of mixed formulation is used for modelling plate bending. The inertia effect of the element is represented by consistent mass matrix.

As the stresses are varying along the thickness, the plasticity should be applied together with elasticity for every iteration. This introduces the concept of "elastic height", which decides the stresses in the plate. And to find out the elastoplastic stiffness matrices, the integration should be done in both elastic core and plastic outer region.

The computer code is first used to solve a standard problem and after validation it was used for dynamic cases of impact loading. The dynamic loading increases the plasticity and in turn the plasticity in the material acts as an dampener to the dynamic oscillatory response. The interdependence of plastic and dynamic behaviours is analyzed in the present work.

1.1 GENERAL INTRODUCTION

Plasticity in the plate is different from a simple two dimension analysis of the plasticity. The main difference is the variation of stress along the thickness. As the first approximation of the plate stress and strain in the thickness direction are neglected [Reissner Mindlin plate formulation]. And if the plate is thin then the out of plane stresses (τ_{xz}, τ_{yz}) are also neglected. For a thick plate in-plane-stresses($\sigma_x, \sigma_y, \tau_{xy}$) varies with the height while out of plane stresses remain almost constant along the height. This variation of the stresses affects the size of the elastic region in the core of the plate.

In general plate formulation is made in terms of lateral forces and moments. In plastic case since the in-plane stresses are different in the elastic core and outer plastic zone, the calculation of the stress resultant moments becomes a function of the size of elastic core. The whole thickness is not plastic, proper weightages are to be given to the elastic and plastic stiffness matrices to calculate effective stiffness matrix. Calculations of unbalances after the integration of dynamic analysis and the plastic analysis is to be done to incorporate both the aspects properly. These taken together increases the complexity of the problem.

1.2 LITERATURE SURVEY

The thin plate theory developed by Kirchhoff is modified for thick plates by Reissner and Mindlin by relaxing the assumption that the normals to the midplane remain normal. This was done since the shear force terms can not be neglected in many loading and support conditions. Reissner and Mindlin theories can be

extended to thick plates and thus known as Reissner-Mindlin formulation. Regarding Finite Element Method a difference between thin plate and thick plate analysis is that in former it is possible to represent the state of deformation by one quantity deflection w and thus C^1 continuity of w is needed, while in latter w can have C^0 continuity along with rotation and shear degree of freedom.

While analytical solutions remain difficult for thick plates, the advent of Finite Element Method made thick plate theory simpler to implement. Thick plate theory on the imposition of additional assumptions behaves as thin plate theory. It is more natural to start with thick plate formulation and make transition to thin plate theory by imposition of certain relation between degrees of freedom [Zienkiewicz and Taylor,1991]. In thick plate formulation instead of one approximate function for w (for calculation of rotational angle), independent interpolation functions for deflection w and rotation θ are taken. During formulation, if the shear forces are eliminated, the formulation is known as "irreducible formulation" and when functions for shear forces are also retained the formulation is known as "hybrid formulation". The latter formulation gives more degree of freedom and so it is preferred to work with in most of the cases [Zienkiewicz and Taylor,1991]

Another aspect is the selection of the element to perform satisfactorily for thick plates as well as thin plates. Many elements are attempted to avoid the problems of locking and singularity but the results of "triangular robust element" are found favourable in this regard [Zienkiewicz and Lefebvre,1988].

For determination of plasticity three things are required: an yield criteria, flow rule and plastic stress strain law. A

fundamental difference between plastic and elastic analysis is that in elastic solution total stress can be evaluated from the total strain alone, while in the plastic response evaluation of stress strain at any time depends on the stress strain history. This depends on the experimental results which are termed as yield criteria. Commonly used yield criterion are continuous one but piecewise linear yield criterion can also be used [Giulio Maier et al.,1987].

Yield criterion defines the onset of plasticity and flow rule governs the relationship between stress and plastic strain increment after the yield point. Flow rule requires the definition of a potential which is taken to be the same as yield surface for the case of associated plasticity. The plastic stress strain law has been defined for nonlinear, linear and perfectly plastic cases [Yamada et al., 1968]. In latter two cases the total stress strain curve is bilinear and in former case usually some power law satisfying the experimental results is defined. Flow rule, thus can be used in plastic cases by forming a plastic potential and then defining the plastic strains by the same. When the plastic potential surface is the same as the yield surface, the plasticity is termed as "associated" otherwise the plasticity remains "unassociated". The flow rules for loading and unloading are not same, and that makes the problem difficult.

Out of many iterative schemes to solve the above non-linear problem, Newton-Raphson method is found favourable in terms of ease in mathematical formulation and in computational effort [Simo and Taylor, 1986]. In Newton-Raphson method, incremental strain is multiplied to plastic stress strain matrix to get the stresses, while in Runge-Kutta method the incremental strain is subdivided in many subincrements then individual subincremented

strains are multiplied to their corresponding stress strain matrices to get subincremented stress. These stresses are added up to get incremental stress. This is known as "subincrementation" [Nayak,1971; Bathe,1990].

For plasticity in the Mindlin plates, it is suggested to use full three dimensional analysis of stress field. Inclusion of shear terms in the yield function allows the spread of plasticity from the extreme fibre to entire plate thickness [Hinton and Owen,1980]. A very thick plate can be divided in many layers of thin plates and with the Mindlin plate theory,the problem is solved [Hinton and Owen,1980]. But the triangular robust element is proved favourable for transition from thick to thin plates, and the non layered approach can be used for moderately thick plates [Zienkiewicz and Lefebvre,1988]. Elastoplastic numerical experiments has been carried out adopting a full three dimensional version and different criteria of yield in the past [Hinton and Owen,1984].These results show excellent agreement in static elastic analysis and the prediction of vibrational frequencies and buckling loads. These results forms the basis of comparison for the present work.

In elastodynamic, a detailed analyses of various time integration schemes from point of view of accuracy and speed, were reported by [Seron et al.,1990; Zienkiewicz and Taylor,1991; Bathe,1990]. It is concluded that,even though central difference scheme is the best in terms of computational speed and cost, Newmark's constant average scheme gives better accuracy and offers unconditional stability. Whereas, the time step size has negligible effects when central difference scheme is employed [Bazant and Celep,1983].

In elastoplastic dynamic analysis based on Mindlin plate, the work started quite late. Most of the work is in the area of concrete structure under dynamic loading [Kujawski et al.,1984].

Experimental information on material behaviour is scarce for most structural materials [Hinton and Owen,1980]. Instantaneous yield stress is significantly influenced by the rate of straining, also the value of elasticity modulus is found to be dependent on the strain rate for structural material those varies from the materials of limited ductility to brittle elastic behaviours. For this purpose a better understanding of observed phenomenon and underlying microscopic behaviour is required [Hinton and Owen, 1980].

In loading and unloading the yield point of the material changes because of strain hardening. A model for yield limit degradation has been developed by Bazant [1987]. Other study [Panzeca et al.,1987] deals with a transformation matrix that takes care of yield point variation according to the loading and history.

Application of numerical model to dynamic flow of steel specimen has been done by Casadei et al.[1987]. The results compared favourably with the experimental result. For beams and rods a code validation case study is done by Marques [1987]. An elasto-plastic one degree of freedom numerical model was presented for steel slab under blast loading [Mazza et al.,1987]. The results were compared with static elastoplastic loading. Also both of the results were compared with experimental data. It was found out that in the dynamic loading conditions the displacements were intensified at the plastic zone compared with those of static loading. In the area of explosive welding also

the study was carried out [Ante, 1987]. In this case welding of two concentric tubes were considered while explosive was put inside the inner tube. The specimen experienced both mechanical as well as thermal shock. In this problem plasticity was induced by thermal load also.

PRESENT WORK

The present work deals with the study of the effect of plasticity in plates under various type of loadings both static and dynamic.

A mixed finite element analysis using Reissner Mindlin plate theory is used to model the plate bending. To avoid locking and singularity a triangular robust element (T6/3 B3) [Zienkiewicz and Taylor, 1991] is used.

Chapter 2 deals with the theoretical part of the formulation of plate bending and plasticity. Chapter 3 deals with the finite element implementation of the plastic analysis and its integration with dynamic analysis. Chapter 4 describes the validation of the code and the behaviour of plasticity in dynamic loading and effect of plasticity on the dynamic response. Chapter 5 presents the conclusions and the suggestions for future work.

CHAPTER 2

BASIC EQUATIONS OF ELASTOPLASTIC DYNAMIC PLATE ANALYSIS

2.1 INTRODUCTION

This chapter presents the basic equations governing the plate under elastoplastic and dynamic conditions. As explained in earlier chapter, by appropriate element selection and formulation Mindlin plate theory can be used for thick and thin plates.

The Mindlin plate theory makes the following assumptions,

1. The strain and the stress components in Z-direction are negligible (σ_z and $\epsilon_z \approx 0$) (fig 2.1), where Z-direction represents the thickness direction.
2. The normals to the middle plane of the plate remains straight during deformation.

Applying above assumptions, static equilibrium equations for plate bending problem can be obtained [Timoshenko,1959] in terms of midplane rotations(θ_x,θ_y),lateral displacements(w) of the midplane and the shear forces(S_x,S_y) at the cross-section. Sign convention and the variables are shown in fig(2.1).

Stress resultant moments and shear forces are defined in terms of stress variables as follows:

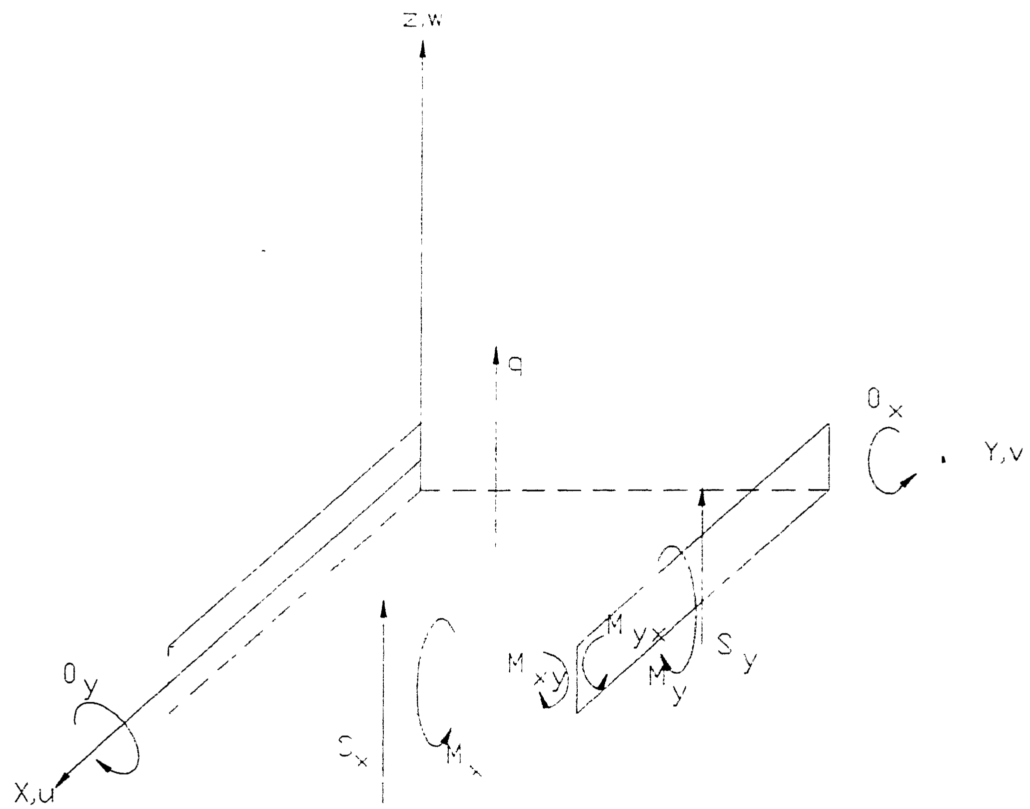


FIG 2.1

$$M_x = \int_{-h/2}^{h/2} \sigma_x \, z \, dz \quad , \quad M_y = \int_{-h/2}^{h/2} \sigma_y \, z \, dz \tag{2.1a}$$

$$M_{xy} = \int_{-h/2}^{h/2} \tau_{xy} \, z \, dz \tag{2.1b}$$

$$S_x = \int_{-h/2}^{h/2} \tau_{xz} \, dz \quad , \quad S_y = \int_{-h/2}^{h/2} \tau_{yz} \, dz \tag{2.1c}$$

For the dynamic analysis inertia forces due to mass and moment of inertia are considered in equilibrium equations. Formulating equilibrium equation using d' Alembert's principle, the following equations are obtained.

$$\frac{\partial M_x}{\partial x} + \frac{\partial M_{xy}}{\partial y} - S_x - \rho \frac{h^3}{12} \ddot{\theta}_x = 0 \tag{2.2a}$$

$$\frac{\partial M_{xy}}{\partial x} + \frac{\partial M_y}{\partial y} - S_y - \rho \frac{h^3}{12} \ddot{\theta}_y = 0 \tag{2.2b}$$

$$\frac{\partial S_x}{\partial x} + \frac{\partial S_y}{\partial y} + q - \rho h \ddot{w} = 0 \tag{2.2c}$$

Strains at a height of z from the centre plane are given as,

$$\epsilon_x = z\frac{\partial\theta_x}{\partial x} \quad , \quad \epsilon_y = z\frac{\partial\theta_y}{\partial y} \tag{2.3a}$$

$$\gamma_{xy} = z\left[\frac{\partial\theta_x}{\partial y} + \frac{\partial\theta_y}{\partial x}\right] \tag{2.3b}$$

$$\gamma_{xz} = \theta_x + \frac{\partial w}{\partial x} \quad , \quad \gamma_{yz} = \theta_y + \frac{\partial w}{\partial y} \tag{2.3c}$$

2.2 THE MIXED FINITE ELEMENT FORMULATION

The finite element formulation involves three sets of variables θ, S, w. These variables at any point within the element are expressed in terms of nodal parameters($\theta_x, \theta_y, S_x, S_y, w$) of the element using shape functions.

$$\theta = \begin{bmatrix} \theta_x \\ \theta_y \end{bmatrix} = N_\theta \overline{\theta} \tag{2.4a}$$

$$S = \begin{bmatrix} S_x \\ S_y \end{bmatrix} = N_s \overline{S} \tag{2.4b}$$

$$w = N_w \overline{w} \tag{2.4c}$$

Where θ, S, w are the nodal values of respective variables.

Now using eq(2.3) and eq(2.1) the eq(2.2) can be rewritten as follows.

$$L^{T}DL\theta - S - \rho\frac{h^3}{12}\ddot{\theta} = 0 \qquad (2.5a)$$

$$\theta - C_1 S + \nabla w = 0 \qquad (2.5b)$$

$$\nabla^T S + q - \rho h\ddot{w} = 0 \qquad (2.5c)$$

Where L,∇ operators and C_1 matrices are given by,

$$L = \begin{bmatrix} \frac{\partial}{\partial x} & 0 \\ 0 & \frac{\partial}{\partial y} \\ \frac{\partial}{\partial y} & \frac{\partial}{\partial x} \end{bmatrix}, \quad \nabla = \begin{bmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \end{bmatrix}, \quad C_1 = \frac{1}{Gh}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$D = \frac{Eh^3}{12(1-\nu^2)}\begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix}$$

Now by Galerkin method, using eq(2.5) and eq(2.4), and then integrating over the area of the element, the following equations are obtained.

$$A\bar{\theta} + B\bar{S} + E\ddot{\bar{\theta}} = f_\theta \qquad (2.6a)$$

$$B^T\bar{\theta} - P\bar{S} + C\bar{w} = 0 \qquad (2.6b)$$

$$C^T\overline{S} + F\overline{w} = f_w \tag{2.6c}$$

Where,

$$A = \int_{\Omega} (LN_\theta)^T D (LN_\theta)\, d\Omega \quad , \quad B = \int_{\Omega} N_\theta^T N_s d\Omega$$

$$C = \int_{\Omega} N_s^T (\nabla N_w)\, d\Omega \quad , \quad P = \int_{\Omega} N_s^T C_1 N_s d\Omega$$

$$f_\theta = \int_{\Gamma_t} N_\theta^T\, T\, d\Gamma \quad , \quad f_w = \int_{\Omega} N_w^T\, q\, d\Omega$$

$$E = \frac{\rho\, h^3}{12} \int_{\Omega} N_\theta^T\, N_\theta\, d\Omega \quad , \quad F = \rho\, h \int_{\Omega} N_w^T\, N_w\, d\Omega$$

In the above equations the vector T represents two moment components imposed on the boundary by the traction and q is the intensity of the imposed lateral force. The shape functions N_θ, N_w are chosen to have C_0 continuity, but because of shear forces are defined only for inside nodes of the element, N_s can be discontinuous between the elements. Such a choice allows S to be defined locally for each element and thus be eliminated when P is a non-singular matrix. Thus the problem includes only θ and w as variables.

The final formulation in terms of θ and w, after condensing S, becomes :

$$\begin{bmatrix} A + BP^{-1}B^{T} & BP^{-1}C \\ C^{T}P^{-1}B^{T} & C^{T}P^{-1}C \end{bmatrix} \begin{Bmatrix} \overline{\theta} \\ \overline{w} \end{Bmatrix} + \begin{bmatrix} E & 0 \\ 0 & F \end{bmatrix} \begin{Bmatrix} \overline{\theta} \\ \overline{w} \end{Bmatrix} = \begin{Bmatrix} f_{\theta} \\ f_{w} \end{Bmatrix} \tag{2.7}$$

In short eq(2.7) can be written as,

$$[K]^{e}\{u\} + [M]^{e}\{\ddot{u}\} = \{F\}^{e} \tag{2.8}$$

2.3 PLASTIC ANALYSIS

When the stresses go beyond the yield value, then stiffness matrix changes depending on the individual stress component, effective stress and flow rule. In such cases no direct solution can be used, so iterative schemes are used to solve the problem.

To do the non-linear problems, the following steps are used:

a. Calculation of stress and strain

After obtaining the maximum possible elastic solution, the correspondig strains are calculated by the following formula which is obtained by substituting eq(2.3) using the variables from eq(2.4).

$$\{\epsilon\} = \begin{bmatrix} z(LN_{\theta}) & 0 \\ N_{\theta} & \nabla N_{w} \end{bmatrix} \{u\} = [BB]\{u\} \tag{2.9}$$

In a plate problem inplane strains $(\epsilon_x, \epsilon_y, \gamma_{xy})$ and thus

stresses($\sigma_x, \sigma_y, \tau_{xy}$) are the functions of the height, while out of plane strains(γ_{xz}, γ_{yz}) and stresses(τ_{xz}, τ_{yz}) are constant across the thickness. And the stress strain relationship,

$$\{\sigma\} = [DD]\{\epsilon\} \tag{2.10a}$$

$$[DD] = \frac{E}{1-\nu^2}\begin{bmatrix} 1 & \nu & 0 & 0 & 0 \\ \nu & 1 & 0 & 0 & 0 \\ 0 & 0 & \frac{1-\nu}{2} & 0 & 0 \\ 0 & 0 & 0 & \frac{1-\nu}{2k} & 0 \\ 0 & 0 & 0 & 0 & \frac{1-\nu}{2k} \end{bmatrix} \tag{2.10b}$$

Where

$$\{\sigma\} = \begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \\ \tau_{xz} \\ \tau_{yz} \end{bmatrix} , \qquad \{\epsilon\} = \begin{bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \\ \gamma_{xz} \\ \gamma_{yz} \end{bmatrix}$$

In eq(2.10b) k=1.2 is a factor for improving the performance of the analysis, and this accounts for non uniform shear stress distribution. The stress and strain vectors have five above mentioned components.

b. Yield Criteria

In the present analysis Prandtl-Reuss relation is made use to determine the onset of yield.

$$\sqrt{\sigma_x^2 + \sigma_y^2 - \sigma_x\sigma_y + 3\tau_{xy}^2 + 3\tau_{xz}^2 + 3\tau_{yz}^2} = \sigma_{ys} \tag{2.11}$$

c. Fraction Of Plasticity

As the load is increased, it is known, that the outer most fibres become plastic first and the plasticity spreads in to the core and across the plate. At the same time whole thickness is not plastic. So to find above what height the plate goes plastic, the height of elasticity is calculated as(fig 2.1),

$$h_p = \left[\frac{\sigma_{yield}^2 - 3(\tau_{xz}^2 + \tau_{yz}^2)}{\sigma_x^2 + \sigma_y^2 - \sigma_x\sigma_y + 3\tau_{xy}^2}\right]^{-1/2} \frac{h}{2} \tag{2.12}$$

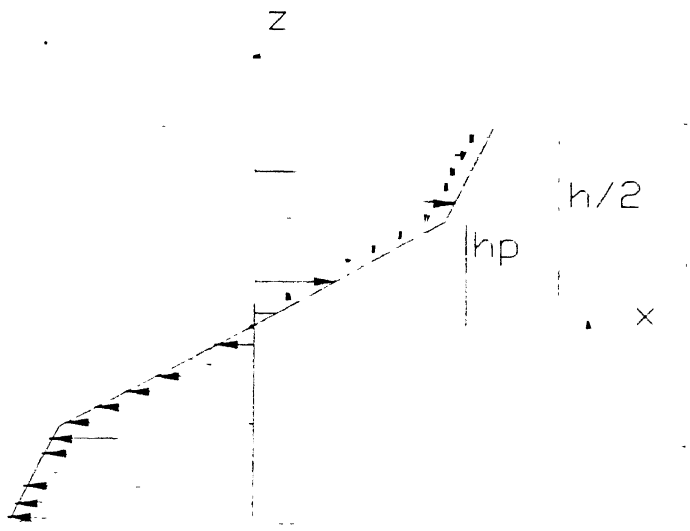
If the numerator of the term inside the root sign goes negative, the whole thickness goes plastic.

d. Flow Rule

Plastic materials are governed by flow rule between stress and plastic strain increment. The flow rule is obtained by the normality principle. The relation between effective stress and strain in simple terms can be understood by the power law in one direction. The power law is an experimental agreement of the material properties in the plastic range.

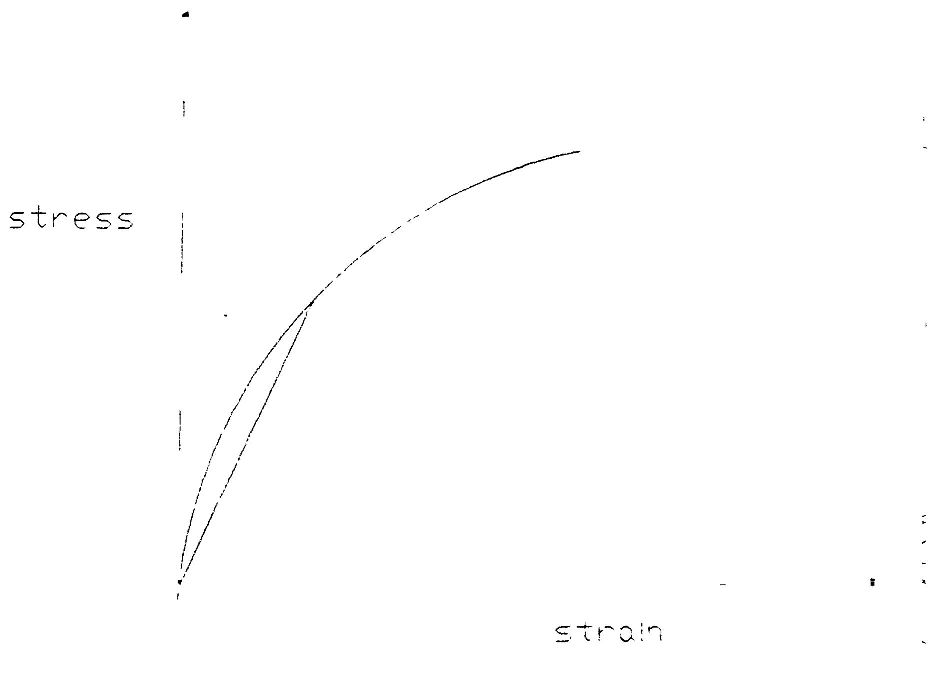
$$\sigma = K\epsilon^n \tag{2.13}$$

Where K and n are material constant and calculated experimentally. The eq(2.13) governs the relation only after yield point. The power law is shown in fig(2.3).



VARIATION OF STRESS ALONG HEIGHT

FIG(2.2)



FIG(2.3)

e. Plastic Stress-Strain Matrix($[DD]^{ep}$)

If a point in the plate goes plastic then the plastic stress strain matrix is used at that point to calculate the incremental stresses from incremental strains in the plastic region.

$$\{\delta\sigma\} = [DD]^{ep}\{\delta\epsilon\} \qquad (2.14)$$

Where

$$[DD]^{ep} = [DD] - \frac{[DD]\{a\}\{a\}^T[DD]}{AA + \{a\}^T[DD]\{a\}}$$

Here [DD] is the property matrix, AA is the slope of stress-strain curve governed by elastoplstic strain stress relation and {a} is the flow vector which depends on the plasticity criteria.

f. Iterative scheme

The unbalance forces to be calculated by commonly used equation,

$$\{f\} = \int [BB]^T\{\sigma\}\,dV \qquad (2.15)$$

does not work in this mixed formulation because of the following reasons.

1. The eq(2.15) can be used if the following expression satisfies the stiffness matrix of an element.

$$[K]^e = \int [BB]^T [DD]\,[BB]\,dV \qquad (2.16)$$

2. Also if the above equation is assembled to form global stiffness matrix for this particular robust element, the global stiffness matrix goes singular.

Answer to these difficulties is found in the mixed formulation which incorporates shear forces also as degree of freedom. Inclusion of such degree of freedom avoids the defects of singularity and locking. Presence of N_s changes the character of stiffness matrix, while the same is not present in eq(2.16).The eq(2.7) is derived from "mixed formulation" while eq(2.15) and eq(2.16) represent the "irreducible formulation".

So leaving eq(2.15) and eq(2.16) aside, another iterative scheme is tried which make use of original elemental stiffness matrix.

For an element, if it goes plastic at a given load then the incremental deformations should satisfy the following equation.

$$[K]^{ep}\{\delta u\} = \{\delta f\} \qquad (2.17)$$

Here $[K]^{ep}$ is the elastoplastic stiffness matrix. Since this matrix is a function of $[DD]^{ep}$ matrix and thus a function of stresses, the global [K] matrix needs updating. So to avoid such recalculations use of the iterative scheme of eq(2.17) is implemented in the following form,

$$[K]^e\{\delta u\} = \{\delta f\} + [[K]^e - [K]^{ep}]\{\delta u\}$$

Now displacement vector on left hand side is treated as displacement vector obtained after n^{th} iteration while the displacement vector on right hand side is the vector before the iteration and is used to calculate unbalance forces. Now for showing the iterative scheme the eq(2.18) can be rewritten as,

$$[K]^e\{\delta u\}_n = \{\delta f\} + [[K]^e - [K]^{ep}]\{\delta u\}_{n-1} \qquad (2.18)$$

The geometrical representation of eq(2.18b) is shown in the fig(2.4).

g. Convergence Criterion

The "error" in each iteration is represented as follows,

$$e = \{u_n\} - \{u_{n-1}\}$$

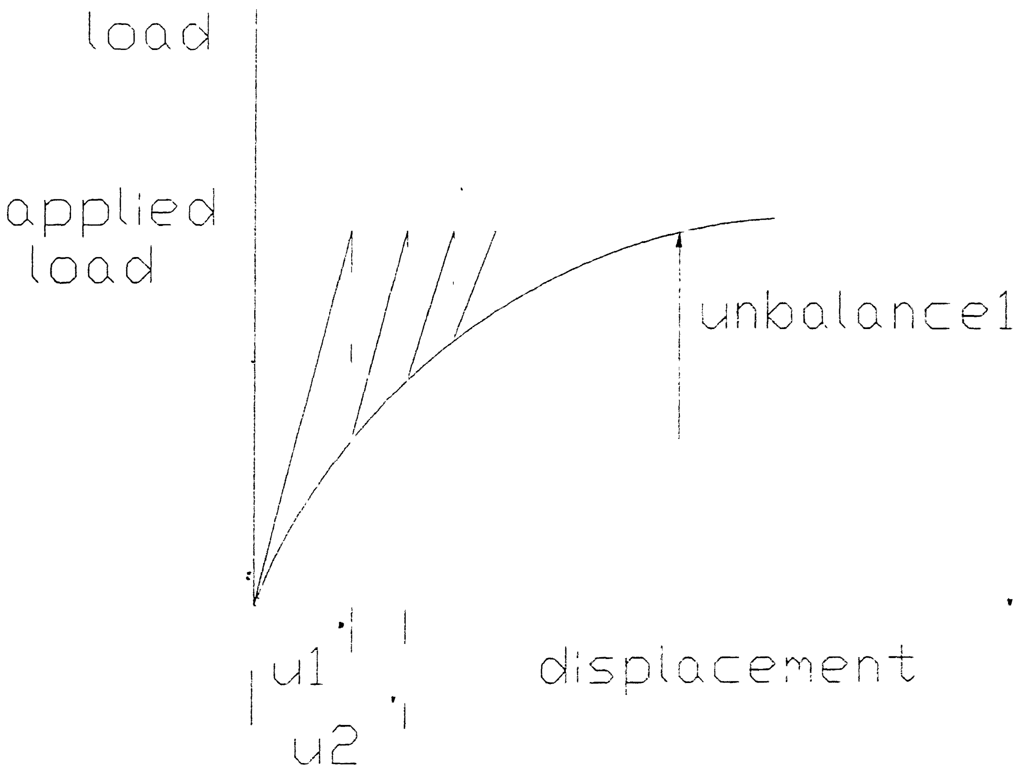
The norm of this error vector will be,

$$|e| = [e^T e]^{\frac{1}{2}}$$

And convergence is said to be achieved when,

$$\frac{|e|}{|u_n|} \leq \alpha \qquad (2.19)$$

Where α is the convergence limit.



FIG(2.4)

h. Effective Plastic Stiffness $[K]^{ep}$ Matrix

$[K]^{ep}$ matrix is the plastic stiffness matrix for an element. Since the whole plate may not go plastic and some portion may still remain elastic, the calculation of this matrix have to made carefully.

Eq(2.14) shows incremental stress strain $[DD]^{ep}$ matrix for full plasticity and eq(2.10b) shows stress strain matrix [DD] for full elasticity. In $[K]^e$ matrix eqn(2.8) the material properties are used at two places in form of [D] and $[C_1]$ matrices. Now matrices [DD] and $[DD]^{ep}$ are defined in the following way.

$$[DD] = \begin{bmatrix} [DD_1] & 0 \\ 0 & [DD_2] \end{bmatrix} , \qquad [DD]^{ep} = \begin{bmatrix} [DD_1]^{ep} & 0 \\ 0 & [DD_2]^{ep} \end{bmatrix}$$

Where $[DD_1]$, $[DD_1]^{ep}$ are 3x3 and $[DD_2]$, $[DD_2]^{ep}$ are 2x2 matrices. Now effective plastic stress strain matrix can be written in the following way.

$$[D]^{ep} = \sum_{j=1}^{7} \left[\frac{2h^3}{3} [DD_1] + \left(\frac{h^3}{12} - \frac{2h_p^3}{3} \right) [DD_1]^{ep} \right] w_j \quad (2.20)$$

Which represents the gauss numerical integration with 7 gauss points of the element. $[DD_1]^{ep}$ and h_p are also different for different gauss points.

And

$$[C_1]^{ep-1} = \sum_{j=1}^{7} \left[2h_p [DD_2] + (h-2h_p) [DD_2]^{ep} \right] w_j \quad (2.21)$$

Fitting $[D]^{ep}$ and $[C_1]^{ep}$ in place of $[D]$ and $[C_1]$ in the eq(2.6), The $[K]^{ep}$ matrix in place of $[K]^{e}$ matrix is obtained.

The loading and unloading parts change the characteristic of stress strain curve and the yield stress. This aspect is also included and discussed in next Chapter with other aspects in implementation of the computer code.

CHAPTER 3

IMPLEMENTATION OF FINITE ELEMENT CODE

This chapter explains the salient aspects of numerical method in developing the FEM code, including element selection.

3.1 Element Selection

Triangular elements are easier for discretizing complex geometries. In literature for different types of problems different type of elements are suggested as they should be free from defects of locking and singularity. Also the performance of the element under different type of loadings and boundary conditions should be stable. "Robust triangular element" with suitable nodal variables satisfy the stability conditions.

The choice of the shape functions and variables should be such that the Babuska-Brezzi [Zienkiewicz and Lefebvre, 1988] conditions are satisfied by the system and thus, stability is ensured. The most vital and strictly necessary condition for the non-singularity of the system given by eqn(2.8) is that,

$$\alpha = \frac{n_\theta + n_w}{n_s} \geq 1 \tag{3.1a}$$

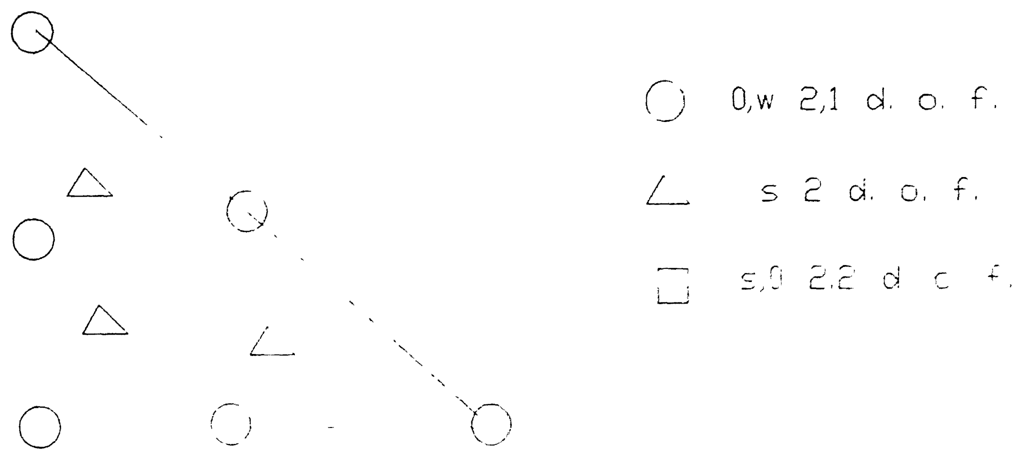
$$\beta = \frac{n_s}{n_w} \geq 1 \qquad (3.1b)$$

Where n_θ, n_s and n_w stand for the number of variables for rotation, shear force and lateral displacements respectively in an element. If either of the two conditions are violated, the locking behaviour will occur in the numerical solution.

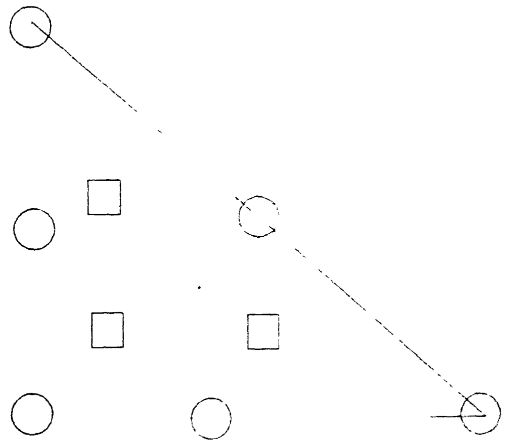
For the dynamic analysis if an implicit time integration scheme is used, it is necessary to use higher order finite elements and a consistent mass matrix. The higher order elements are effective in the representation of bending behaviour. These elements should be employed with consistent load vector, in that the midside and corner nodes are subjected to their appropriate load contribution in the analysis.

In the mixed finite element formulation, triangular element (T6/3) has six boundary nodes which are used to determine a quadratic variation of θ and w [Zienkiewicz and Lefebvre, 1988]. Shear forces are determined by a linear field defined by three internal nodes.

When standard patch tests [Zienkiewicz and Lefebvre,1988;Zienkiewicz et al.,1986] are used with this element, the results indicate that, the element has incipient locking possibility and thus it is not a satisfactory element. But the examination of results indicate that the satisfaction of the requirements can be achieved by the addition of the θ variables in the interior of the element. The three internal



(a) T6/3 (24 d. o. f.)



(b) T6/3 B3 (30 d. o. f.)

FIG. 3.1

nodes created have the shape functions used of the form $L_1^2L_2L_3$, provide a new element (T6/3 B3) which passed the same patch test successfully in all cases [Zienkiewicz and Lefebvre, 1988].

This particular element is chosen for the present analysis. It has 30 degree of freedom, and after the condensation of shear force terms the number of variables reduces to 24.

3.2 Time Integration scheme

The eqn(2.8) is integrated by a numerical procedure. The integration is based on two ideas.

1. Static equilibrium including inertia force is achieved at discrete time intervals Δt.

$$[K]\,^{t+\Delta t}\{u\} + [M]\,^{t+\Delta t}\{\ddot{u}\} = ^{t+\Delta t}\{F\} \tag{3.2}$$

2. Variation of displacement, velocity and acceleration within each time step is assumed. This assumption decides the accuracy and stability of the solution procedure.

The idea no. 2 decides the time integration scheme. The choice of a scheme depends on the finite element idealization. However, the finite element idealization to be chosen depends in turn on the actual wave velocity of the medium to be analyzed. It follows, therefore the selection of an appropriate finite element idealization of a problem and the choice of effective integration scheme for the response solution are closely related and must be considered together.

Out of many schemes "Newmark's implicit scheme, is most

accurate and unconditionally stable.

3.2a Newmark Implicit Scheme

The Newmark's constant acceleration scheme is written as follows [Bathe,1990],

$$^{t+\Delta t}\dot{U} = \ ^{t}\dot{U} + [(1-\delta)\ ^{t}\ddot{U} + \delta\ ^{t+\Delta t}\ddot{U}]\,\Delta t \qquad (3.3a)$$

$$^{t+\Delta t}U = \ ^{t}U + \ ^{t}\dot{U}t + [(\frac{1}{2}-\alpha)\ ^{t}\ddot{U} + \alpha\ ^{t+\Delta t}\ddot{U}]\,\Delta t^{2} \qquad (3.3b)$$

Where α and δ are parameters that control integration accuracy and stability. Newmark originally proposed an unconditional stable scheme, in which case $\delta = 0.5$ and $\alpha = 0.25$.

Using eqn(3.2) and eqn(3.3) final form can be obtained as,

$$[K+a_0M]\ ^{t+\Delta t}U = \ ^{t+\Delta t}\{F\} + [M]\,(a_0\,^{t}U + a_2\,^{t}\dot{U} + a_3\,^{t}\ddot{U}) \qquad (3.4a)$$

$$^{t+\Delta t}\ddot{U} = a_0\,(^{t+\Delta t}U - \,^{t}U) - a_2\,^{t}\dot{U} - a_3\,^{t}\ddot{U} \qquad (3.4b)$$

$$^{t+\Delta t}\dot{U} = \,^{t}\dot{U} + a_6\,^{t}\ddot{U} + a_7\,^{t+\Delta t}\ddot{U} \qquad (3.4c)$$

Where,

$$a_0 = \frac{4}{\Delta t^2} \quad , \quad a_2 = \frac{4}{\Delta t} \quad , \quad a_3 = 1$$

$$a_6 = \frac{\Delta t}{2} \quad , \quad a_7 = \frac{\Delta t}{2}$$

The implicit scheme, as can be seen from the final formulation requires inversion of $[K+a_0M]$ matrix. For this purpose Cholesky decomposition technique is computationally efficient.

3.2b Time step

After deciding on time integration scheme, it is necessary to use a time step suitable with finite element mesh. If the critical wavelength in the medium is denoted by L_w, then the time step can be decided as,

$$\Delta t = \frac{L_w}{c\, n}$$

And the effective length of a finite element should be

$$L_e = c\, \Delta t$$

Where c is wave speed and 'n' is the number of nodes per critical wavelength.

The effective wavelength and corresponding time step must be able to represent the complete bending wave accurately and is chosen appropriately depending on the kind of element idealization and time integration scheme used. It is reported that at least 8 nodes per shortest wavelength are required in order to produce all artifacts of wave propagation [Seron,1990].

However, for higher order (parabolic and cubic) continuum elements the time step has to be further reduced, because the interior nodes are stiffer than the corner nodes [Bathe ,1990]. In addition for T6/3 B3 ,the internal θ variables are quartic functions. Also in the plate elements the flexural modes also affect the time step. In the actual computation appropriate time step is to be selected.

3.3 Implementation Of Plastic And Dynamic Code

At any time step the governing equation of the dynamic problem can be defined as eqn(2.8). Mass matrix [M] does not get affected by the plasticity. So for an incremental load the equation can be written in the elastoplastic region.

$$[M]^{e}\{\delta \ddot{U}\} + [K]^{ep}\{\delta U\} = \{\delta f\} \tag{3.5a}$$

Again rewriting the above equation as we did for eqn(2.17), the equation acquires the following form,

$$[M]^{e}\{\delta \ddot{U}\} + [K]^{e}\{\delta U\} = \{\delta f\} + [[K]^{e} - [K]^{ep}]\{\delta U\} \tag{3.5b}$$

Right hand side shows the sum of the original load and the unbalance. The unbalance arises because of the difference in elastic and plastic stress strain matrices. For every time step this equation is balanced to get an equilibrium displacement satisfying the plasticity in the element. Initial unbalance is taken to be zero and then a trial displacement vector is calculated. This process is continued until both side equalize.

The advantage of the eqn(3.5) is that the global stiffness matrix is not calculated repeatedly, and thus saving computer time.

3.4 Loading And Unloading

If a material becomes plastic while loading and then unloaded, it behaves purely elastically. To implement this aspect the history of the load is to be traced to keep track of the strains in the material. Once the material is unloaded the yield point also changes. So in case of plastic loading and unloading the yield point of every gauss point is different. In case of dynamic loading the arrival of plasticity may have two effects on the stability.

1. The displacements are higher than those in elastic case and thus functional aspect of the design gets affected.

2. The plasticity absorbs much energy, thus the impact of dynamic loading may reduce. Also in case of loading and unloading the energy which comes under hysteresis loop gets released, so the damping and stability in the system may increase.

3.5 Computer Implementation

The computer code developed by Dindore [1994] has been modified to take the elastoplastic nature of the material in the present work.

The present work deals with two type of loadings, uniform

distributed loading and moment loading on the boundary. Plastic code first determines the stresses and if these are still in the elastic region, the plastic part is bypassed. Also if ultimate stress is reached, the programme stops. If the load is in plastic region the load above the yield point is divided into 20 equal part and then applied in small steps. The core size of elasticity (h_p) and stresses are updated after each force iteration to save much computer time. The maximum error due to such scheme can be justified in following way.

$$\text{Maximum stress generated} < \sigma_{ult} < 1.5\ \sigma_{yield}$$

$$so,\ stress\ increment < \frac{1.5\sigma_y - \sigma_y}{20} = 0.025\sigma_y$$

So the percentage of elastic height that will be missed due to above mentioned assumption in each force iteration is,

$$\delta h_p < \frac{0.025\sigma_y}{\sigma_y} x100 = 2.5\%$$

This error gets compensated to the extent because of following reasons,

1. Out of plane stresses are not varying with height.
2. Weightages of various gauss points average out the error.
3. Averaging out if the number of elements are more.

Out of seven gauss points any one or more may be elastic or plastic upto any height in the thickness. The results are shown as contour plots to show extent of plasticity on the plate surface.

CHAPTER 4

RESULTS AND DISCUSSIONS

This Chapter is divided into two parts. Former consists of validation of computer code while later analyzes the interdependence of the plastic and dynamic behaviour.

4.1 Validation of code

Validation of computer code is done on following cases,

case 1. Stresses and deflections for the purely elastic problem are calculated for standard cases (Timeshenko,1959). A satisfactorily agreement (within 1.5 %) is observed with theoretical results. Mesh is used as in fig(4.2).

case 2. For qualitative validation, the plate is subjected to different loads and boundary conditions. The spread of plastic zone was observed and the same was more in the cases where either geometric curvature was more or there was a discontinuity in boundary conditions.

case 3. For quantitative validation, a standard problem is taken up (Hinton, 1984). In this a plate fixed in all edges, subjected to a uniform load is considered (fig 4.1). The material and geometric properties are as follows,

Modulus of elasticity, $E = 30000$ MPa

Poisson's ratio, $\nu = 0.30$

Density $\rho = 0.003 \times 10^6$ Kg/m^3

Thickness of plate, $h = 0.20$ m

Side of the square plate, $L = 6.0$ m

slope in plastic region, $K = 0.01 \times E$

Yield stress, $\sigma_y = 30.0$ MPa

Uniformly distributed load for $q = 0.30, 0.35, 0.40$ MN/m^2

The present finite element analysis consists of dividing the square plate by a uniform mesh as shown in fig(4.2). As can be seen quarter plate is also used by applying symmetric conditions at the boundaries.

Fig(4.3) presents the deflection at the centre of the plate as a function of load. The figure shows the results of considering the mesh as a full plate and as quarter plate(which is equivalent to a finer mesh with elements of half size) together with those semi-loof analysis [Hinton and Owen, 1980] and Hinton and Owen,1984. It can be seen, that the present finite element results are in close agreement with semi-loof analysis. Fig(4.4-4.5) show the spread of plastic zone as contours plots at various load levels. At each load level the different contours correspond to percentage of plastic height. These plots also agree well with the results of Hinton (fig 4.4a) except toward peak load (q=0.40) Hinton's results behave more plastically. This happens because of layered approach used by Hinton. An inherent error of 12.5 percent is induced in dividing the thickness into 8 layers. And at peak loads the present results agree with those of Hinton's in this error range.

Thus, it can be seen that the results by the present code agrees well with the available quasi-static elastoplastic results.

4.2 Plastic and dynamic behaviour

This section presents elastoplastic dynamic results obtained by the present approach. The variation of dynamic load is assumed as follows,

$$q = \frac{q_0}{2}\left[1 - \cos\left(\frac{\pi\, t}{RT}\right)\right] \qquad t \le RT \qquad (4.1)$$

For load # 1,

$$q = q_0 \qquad t > RT$$

And for load # 2,

$$q = q_0 \cos\left(\frac{\pi\,(t - RT)}{2\, RT}\right) \qquad t > RT$$

Where RT is the rising time for the load to reach the peak. The forcing functions are shown in fig(4.6). The element size and time steps are choosen in such a way so that the solution remains stable.

To study the interdependence of plastic and dynamic behaviour, the analysis is divided into three parts.

I. The amount of plasticity which is the ratio of effective stress to yield stress in the most plastic gauss point, is shown against time for various rise time (fig 4.7). The load no. 1 is applied for this case. This is compared with the amount of plasticity in static case and also with onset of plasticity in

that element. For this case the amount of plasticity increases as the rise time is decreased while keeping other parameters same. By reducing the rise time the inertial forces play a significant role in inducing more plasticity.

II. For a given rise time the dynamic response is analyzed for purely elastic and elastoplastic cases. The load no. 2 is applied in this case. The deflection is also seen for different loads with same rising time. The deflection at the centre is plotted against time steps. Initial maximum defletion is more than the purely elastic case. But at latter time the delection is less (fig 4.8). For this case the dynamic response in elastoplastic case is more damped than the purely elastic case. The response becomes more dampened as the plasticity increases in the plate. The dissipation of energy in form of hysteresis losses accounts for this damping.

III. The number of gauss points which have gone plastic, are plotted against the time for different loads and given rise time. The load no. 2 is applied in this case (fig 4.9). For this case because of strain hardening the number of elastic gauss points is increasing during unloading. And at the next peak load the number of plastic gauss points is reducing than the previous one.

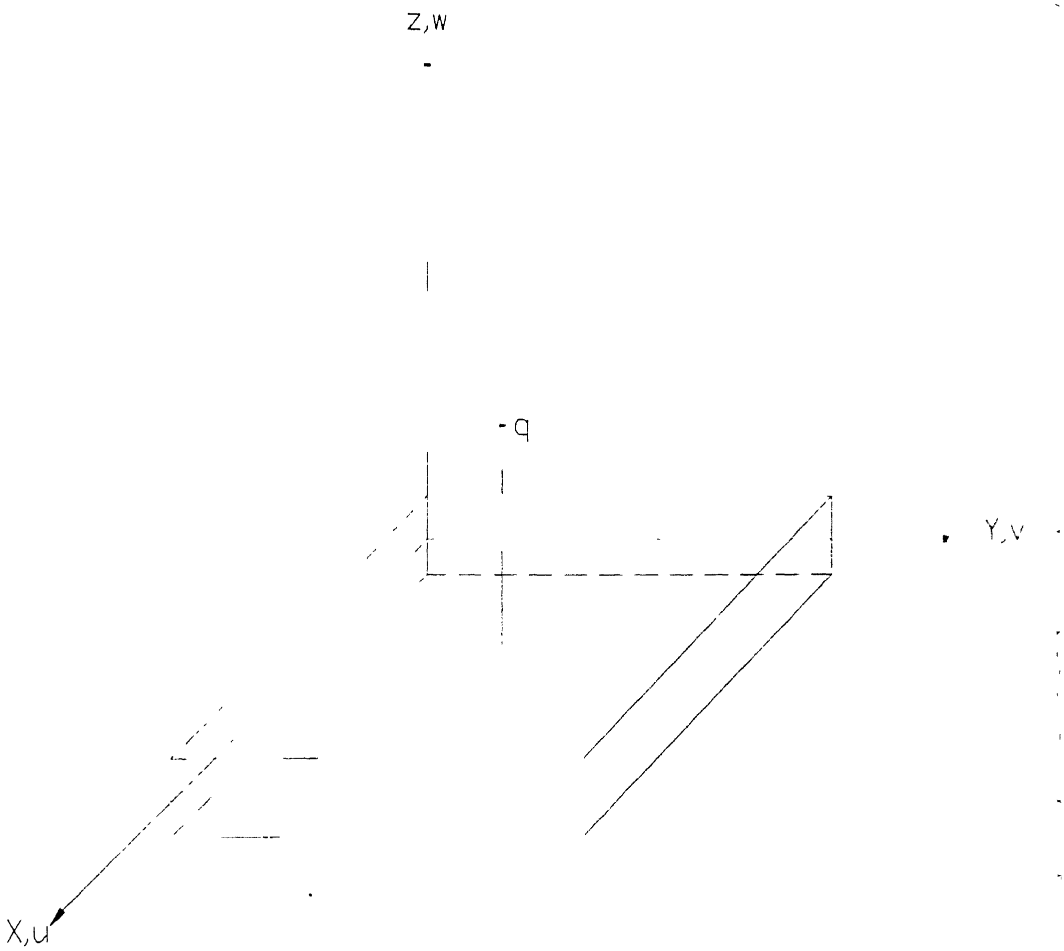
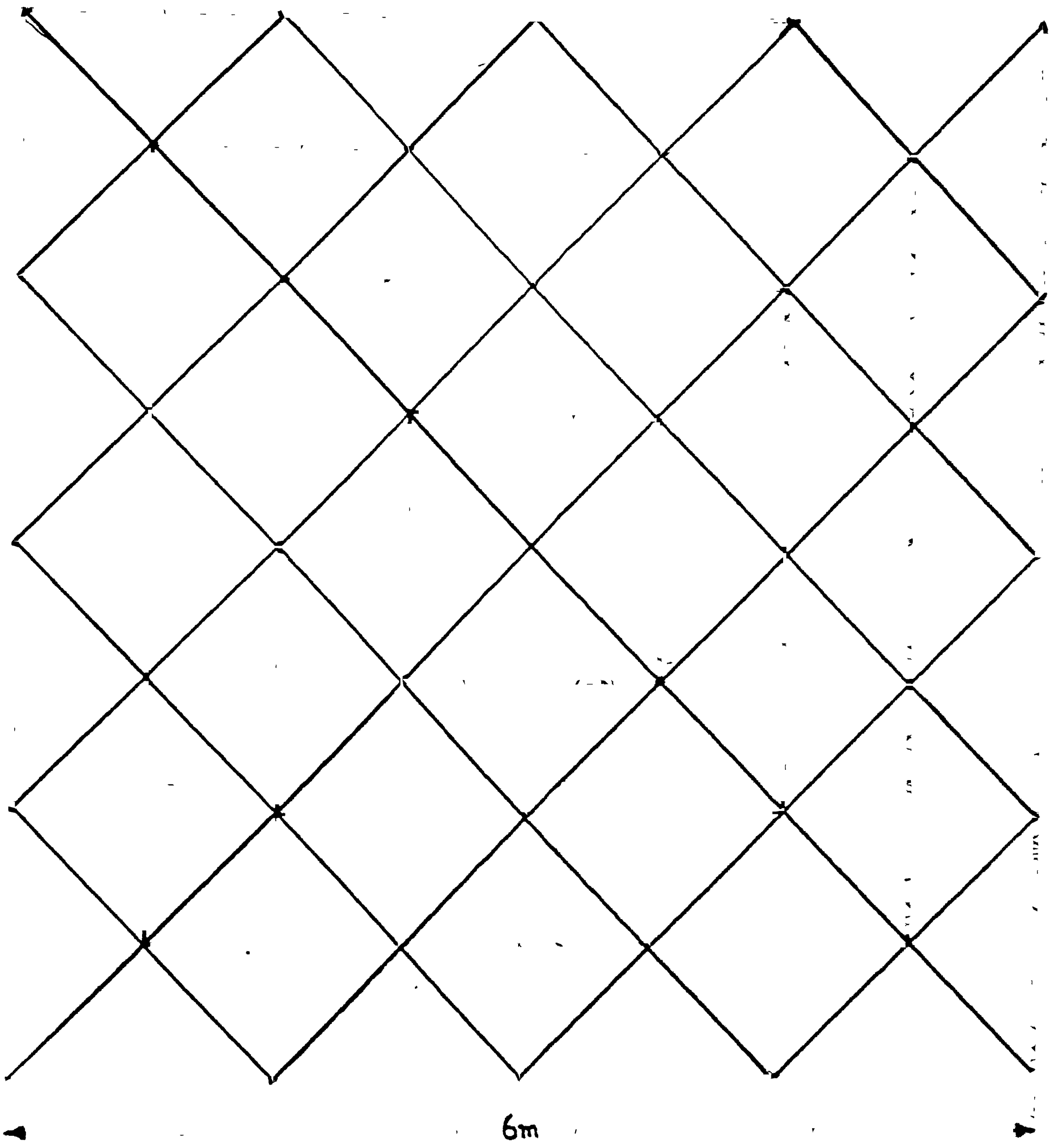


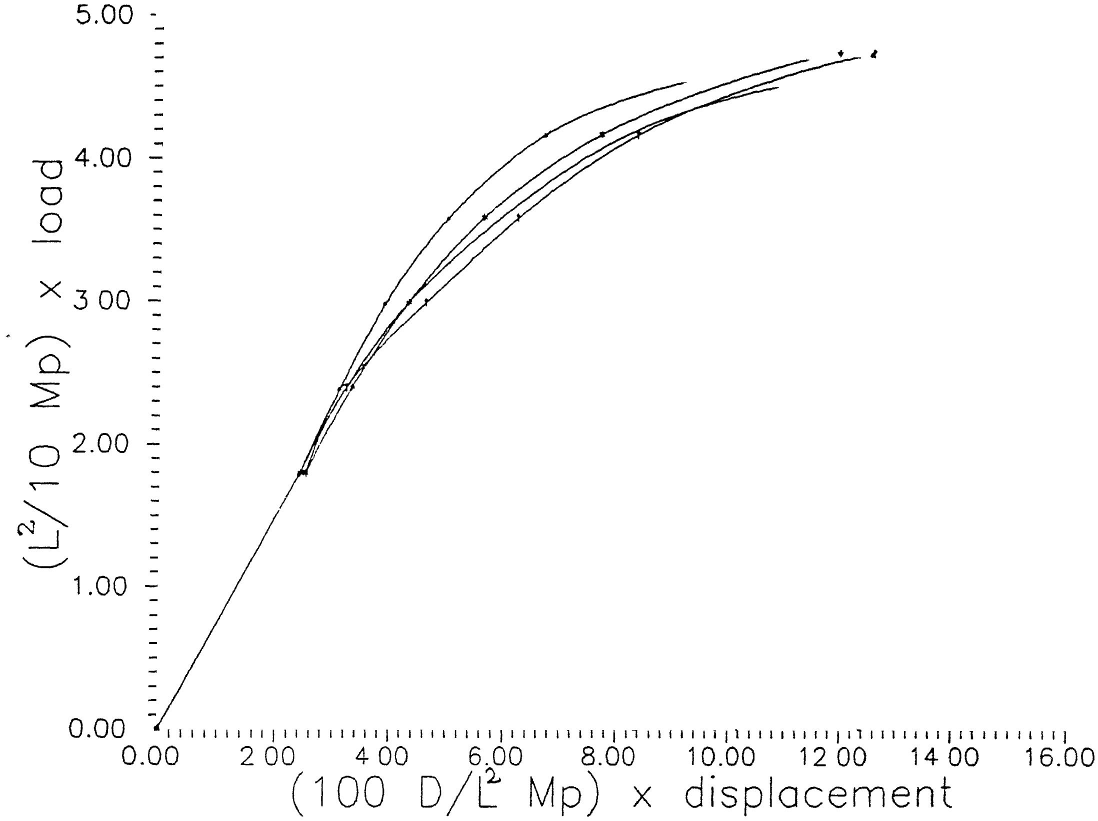
Fig (4.1)

Where q is the uniformly distributed load.

Fig(4.1)



Fig(4.2)



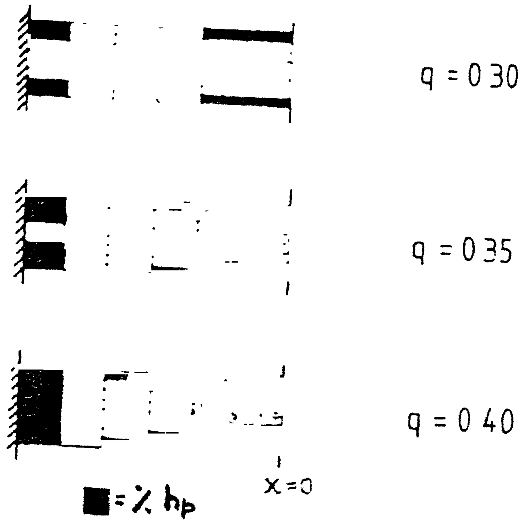
- Semi loof analysis

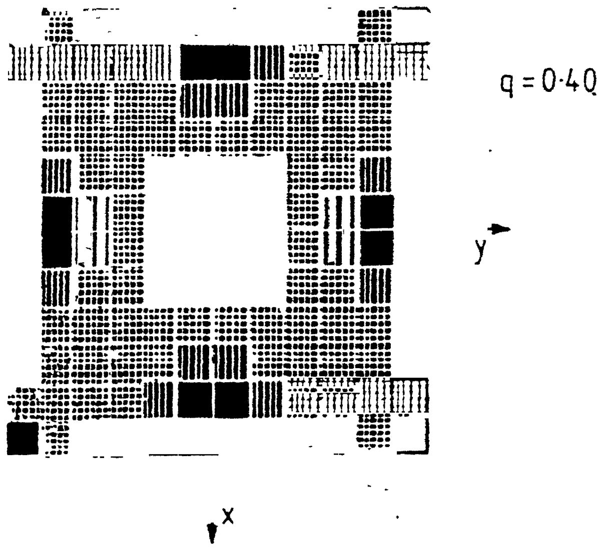
- Hinton's analysis

- Present analysis (full plate)

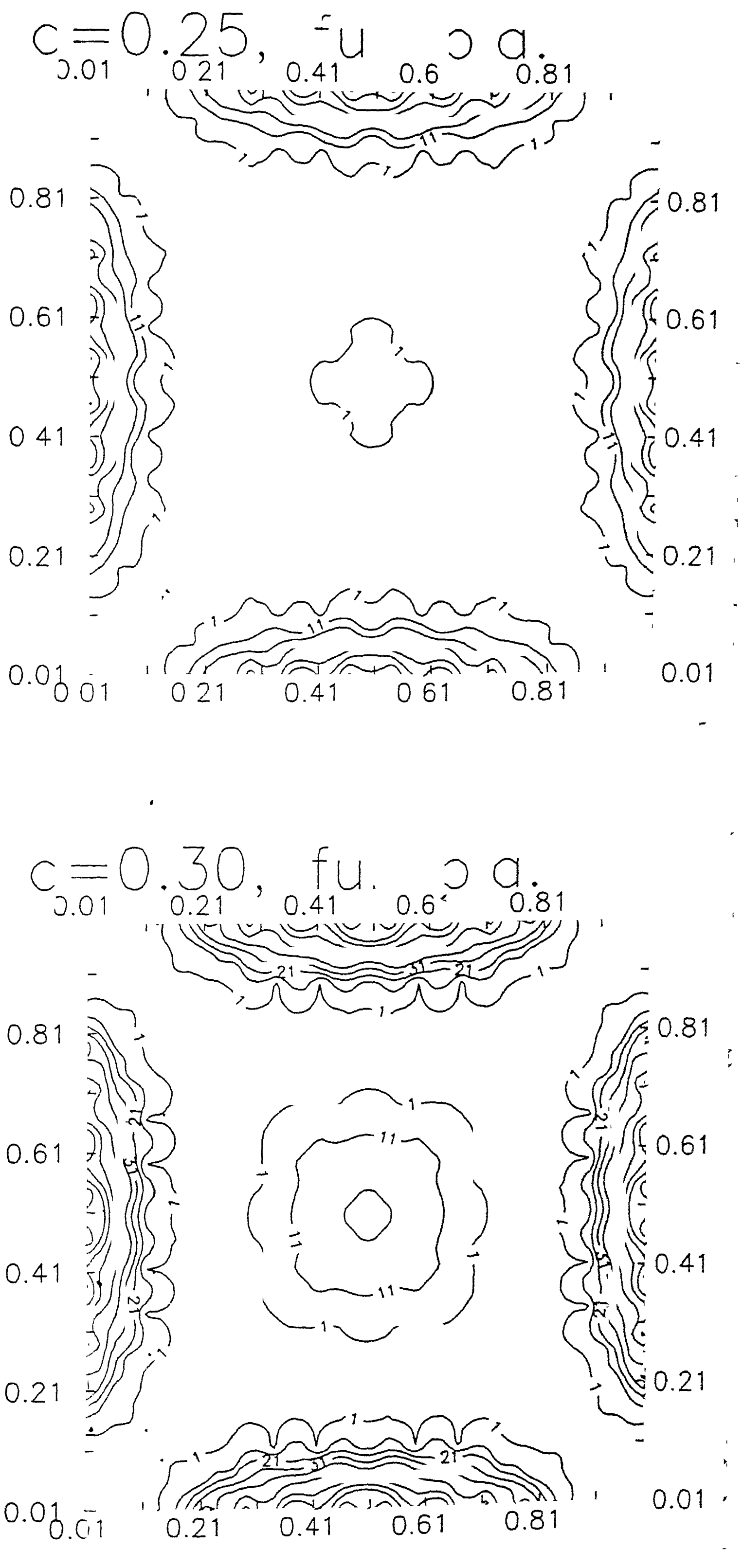
x - Present analysis (quarter plate)

Fig(4.3)

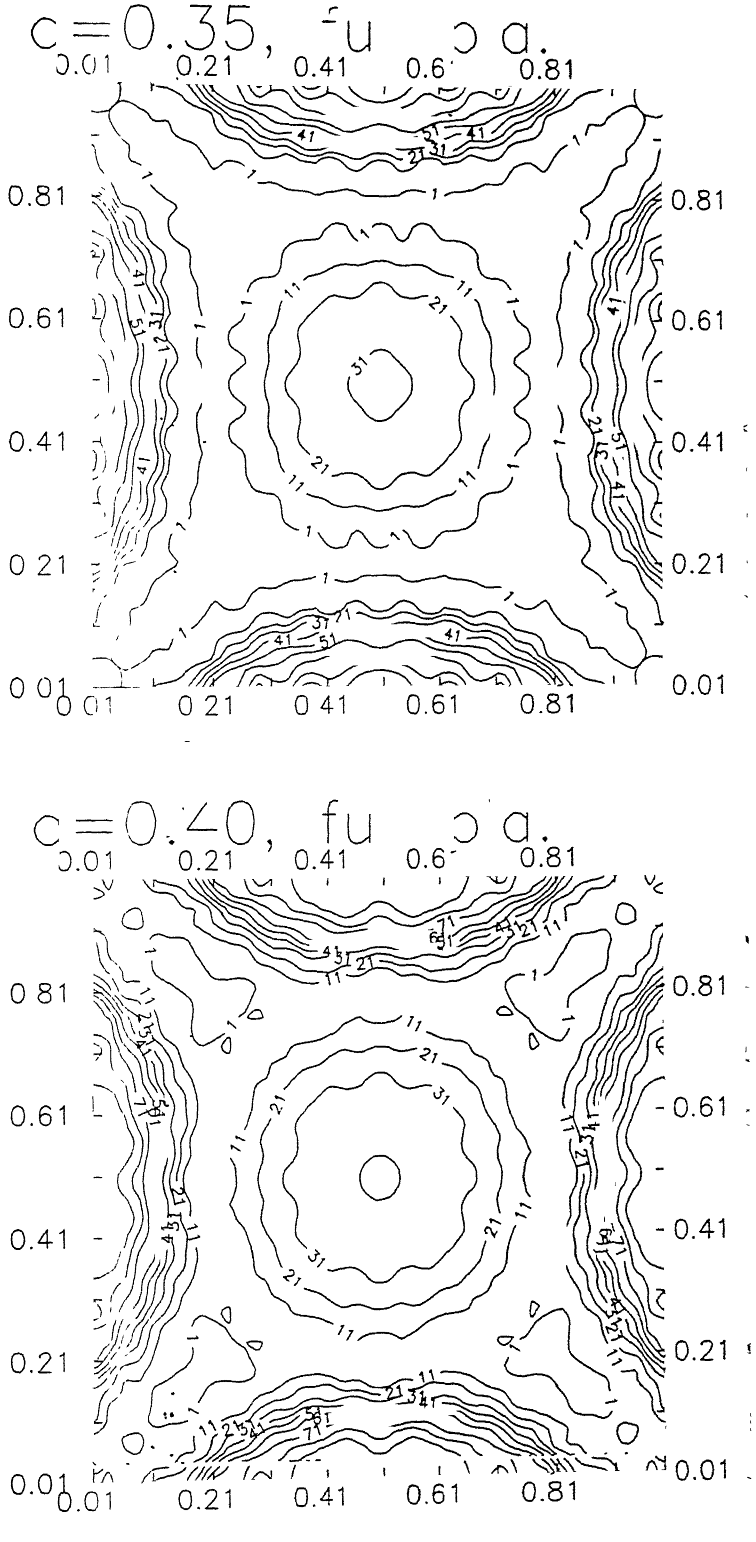




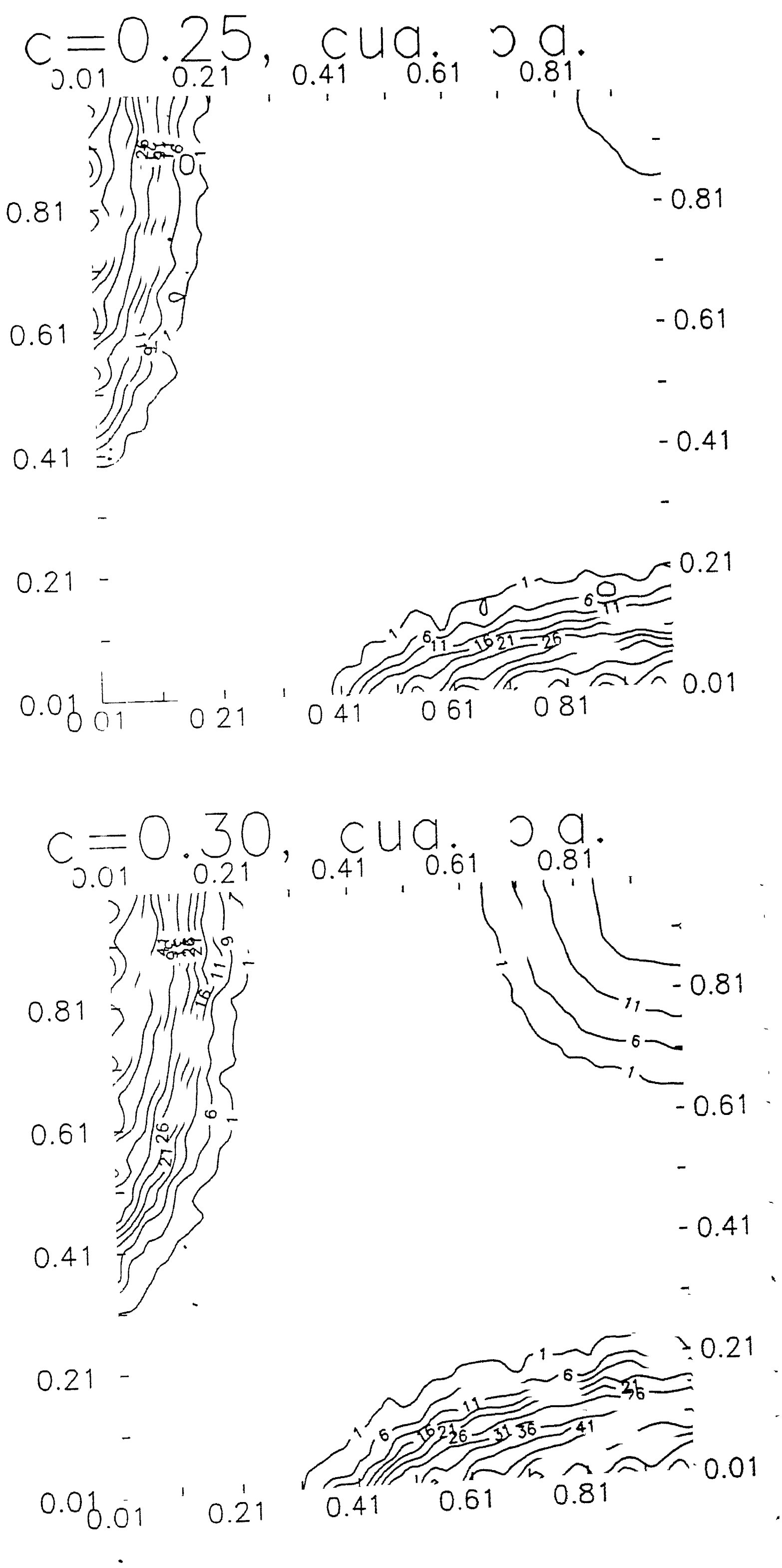
Fig(4.4a): standard results on height and spread of plasticity



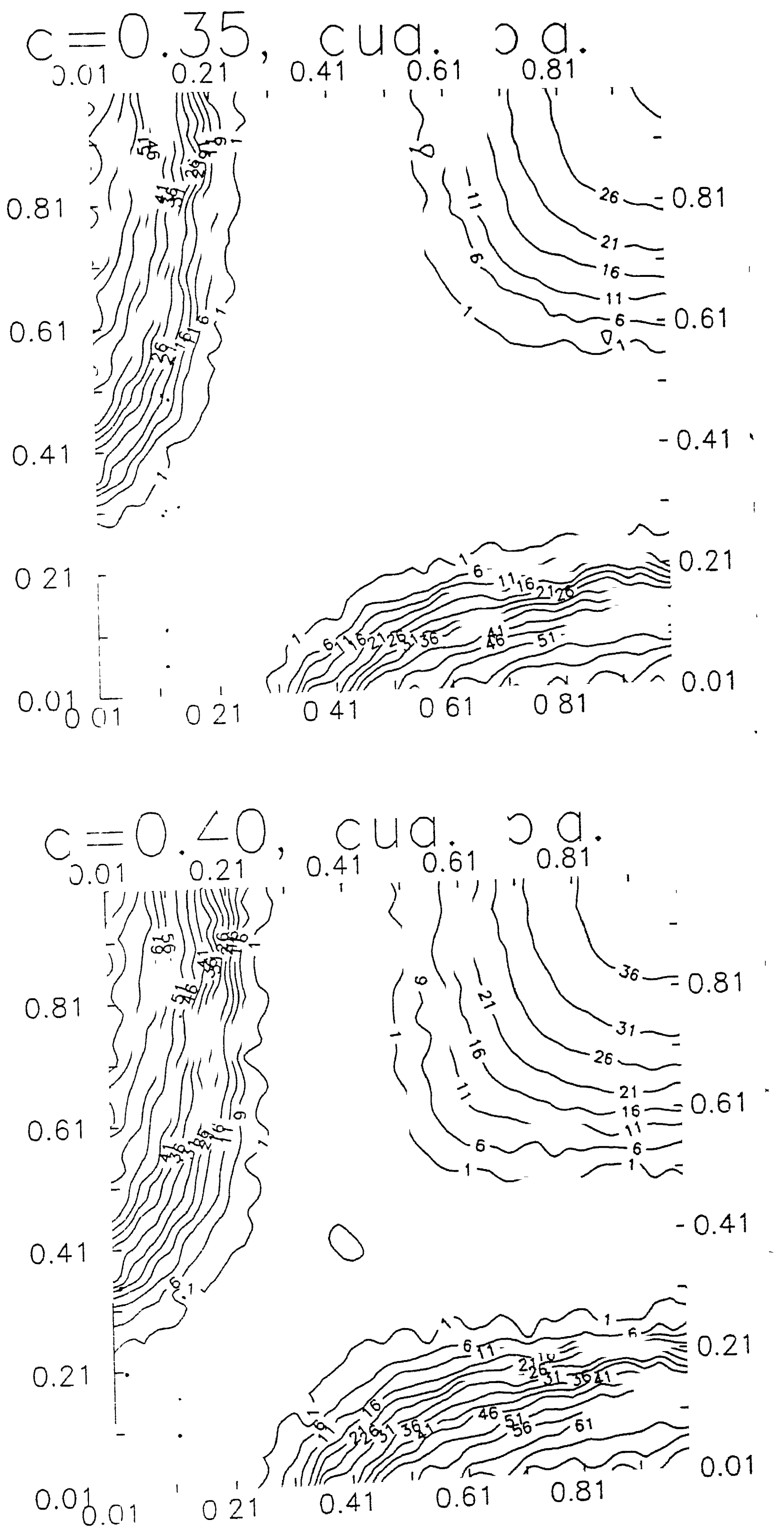
Fig(4.4b): contours show the percentage of plastic height



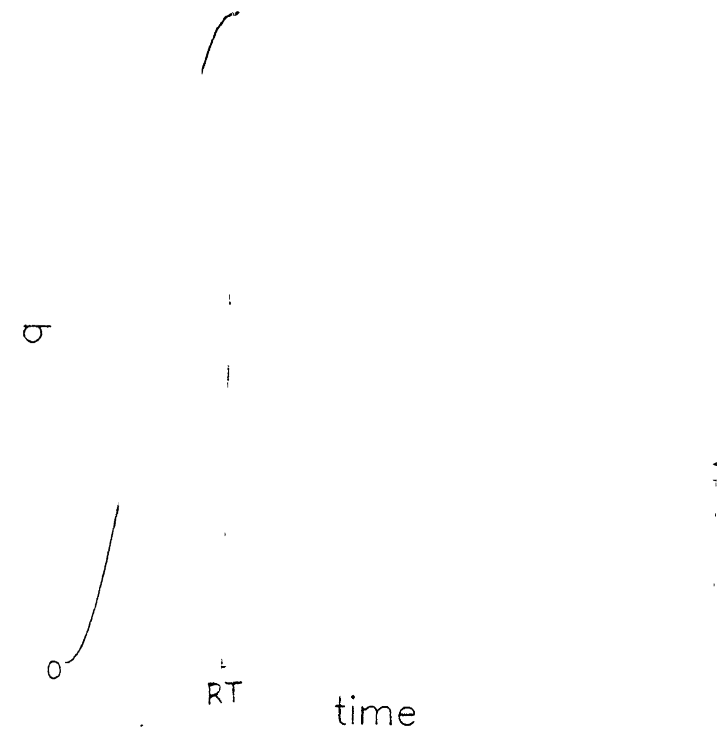
Fig(4.4c): Contours show the percentage of plastic height.



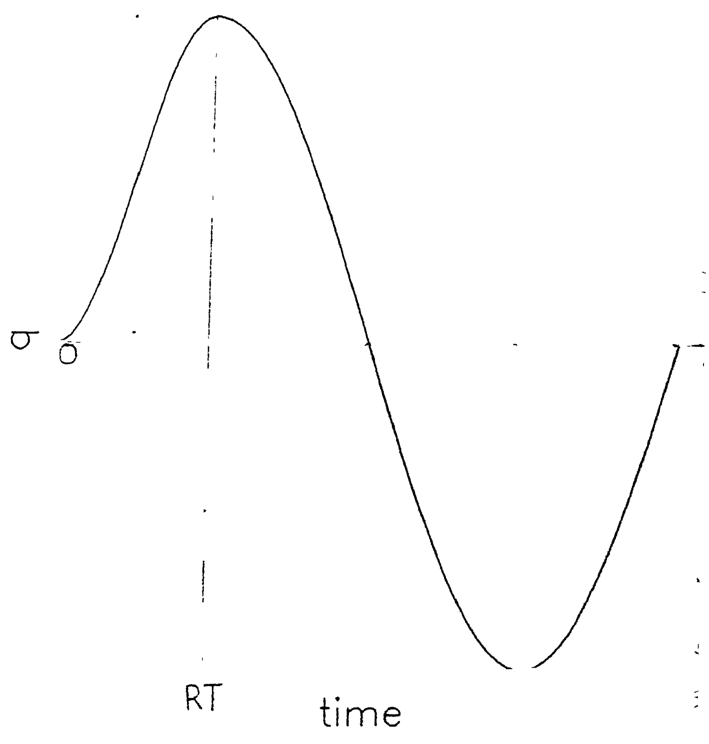
Fig(4.5a): Contours show the percentage of height of plasticity



Fig(4.5b): Contours show the percentage of height of plasticity

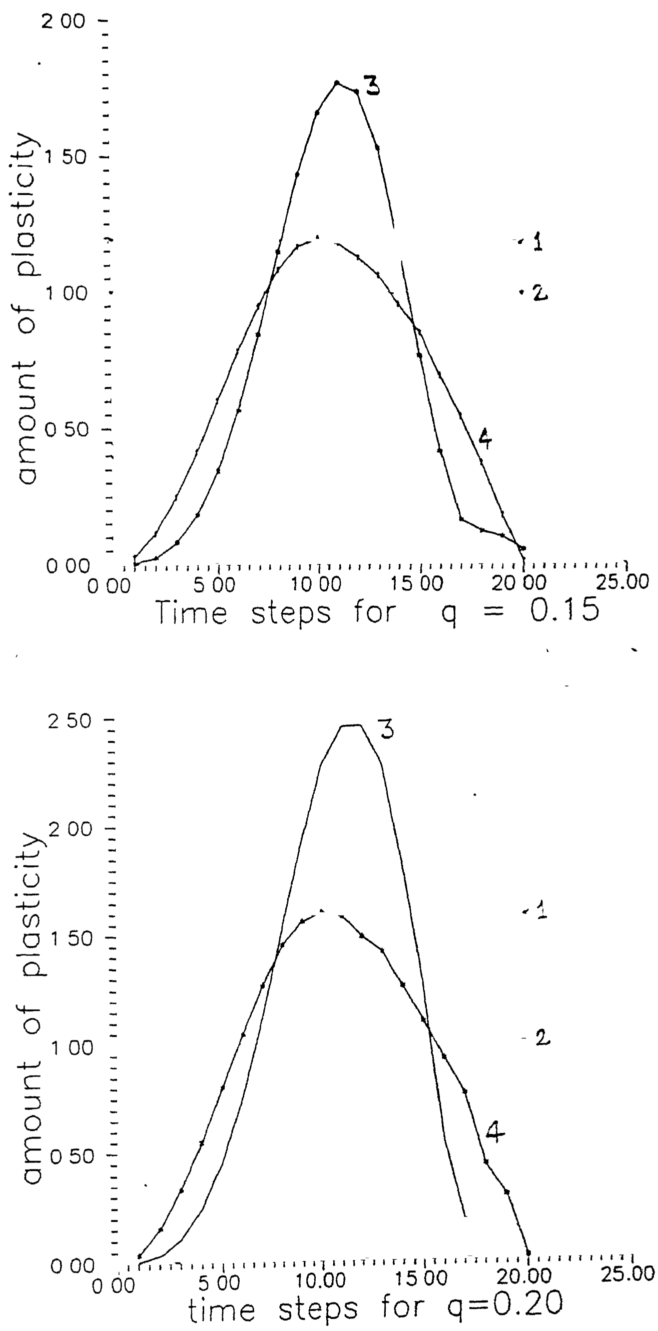


Load no. 1



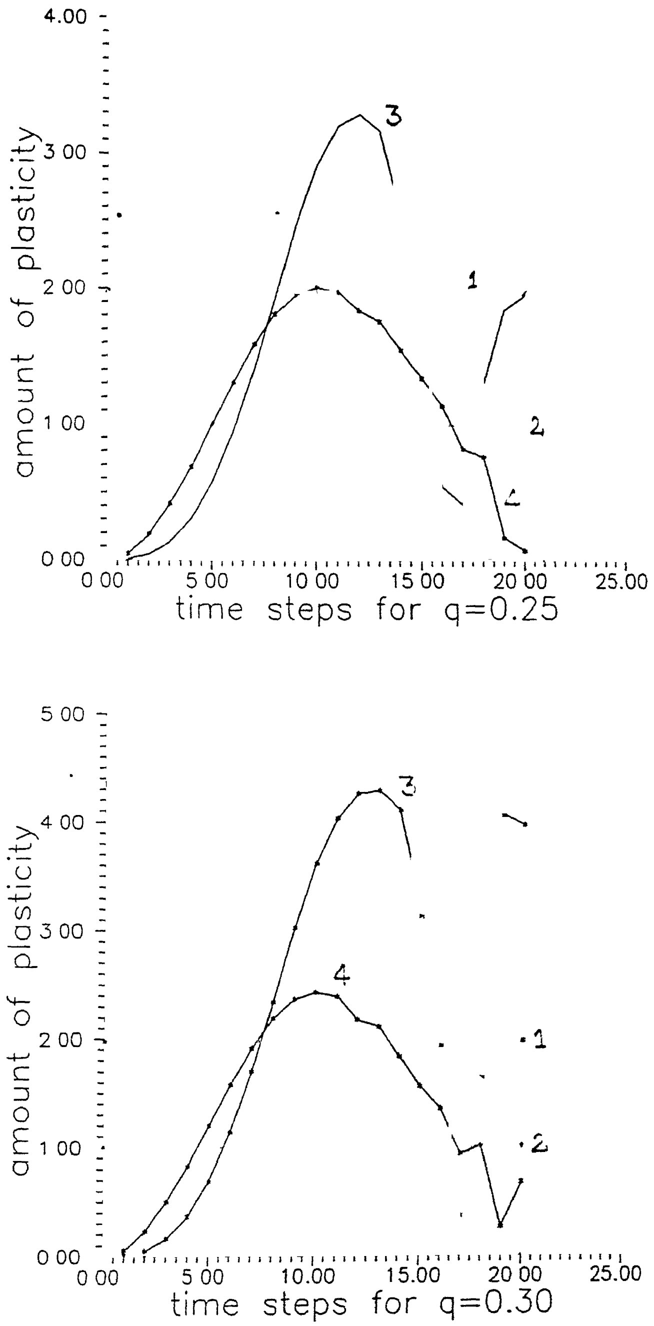
Load no. 2

Fig(4.6): Shows different type of forcing functions.



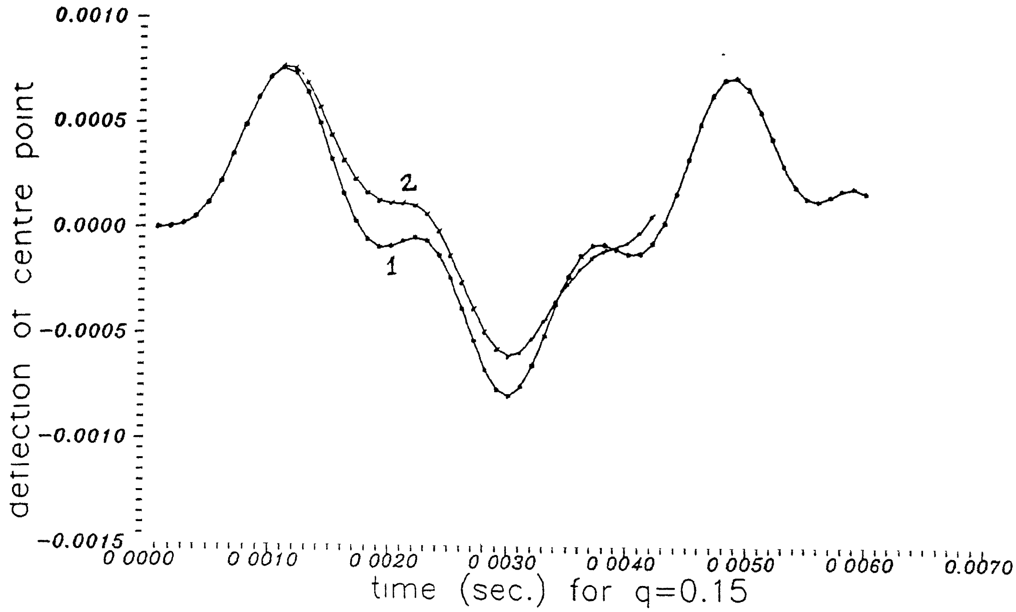
1 - Static case
2 - Onset of plasticity
3 - For rise time = 0.001
4 - For rise time = 0.01

Fig(4.7a): Effect of dynamic loading on the plasticity.

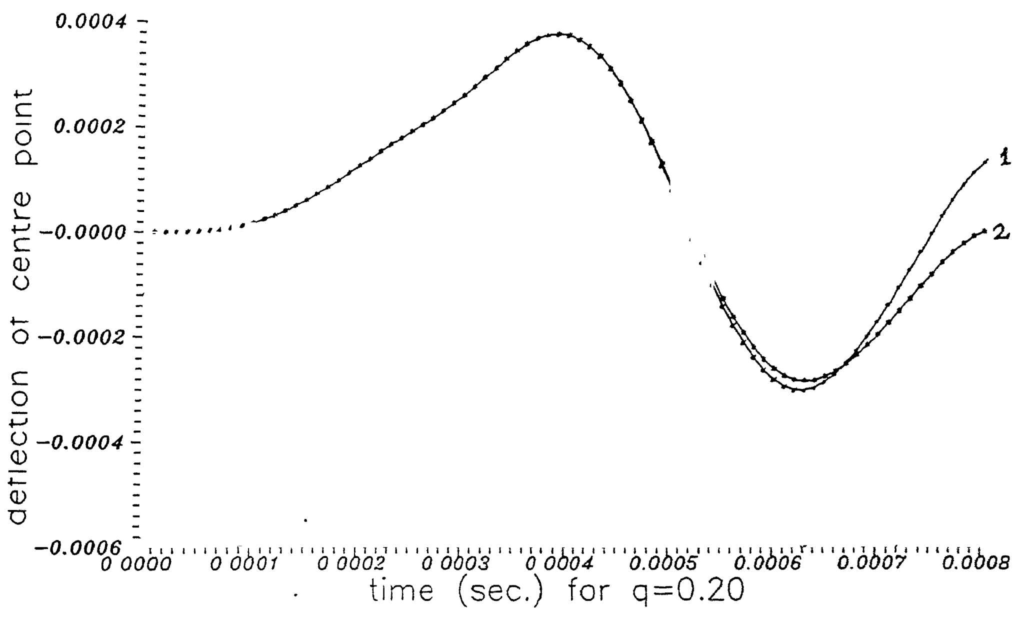


1- Static case
2- Onset of plasticity
3- For rise time =0.001
4- For rise time =0.01

Fig(4.7b): Effect of dynamic loading on plasticity.



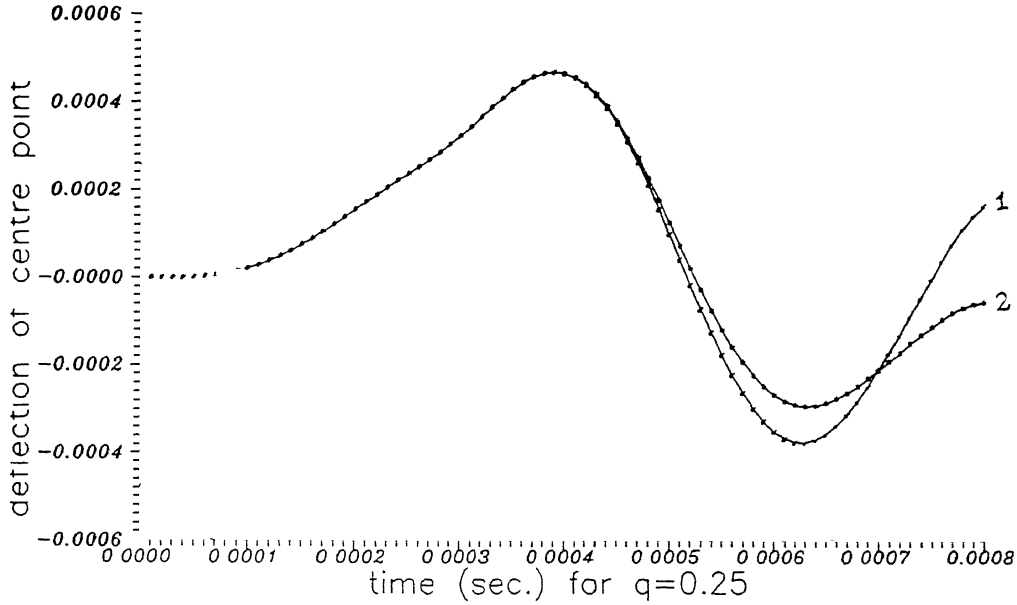
RT=0.001



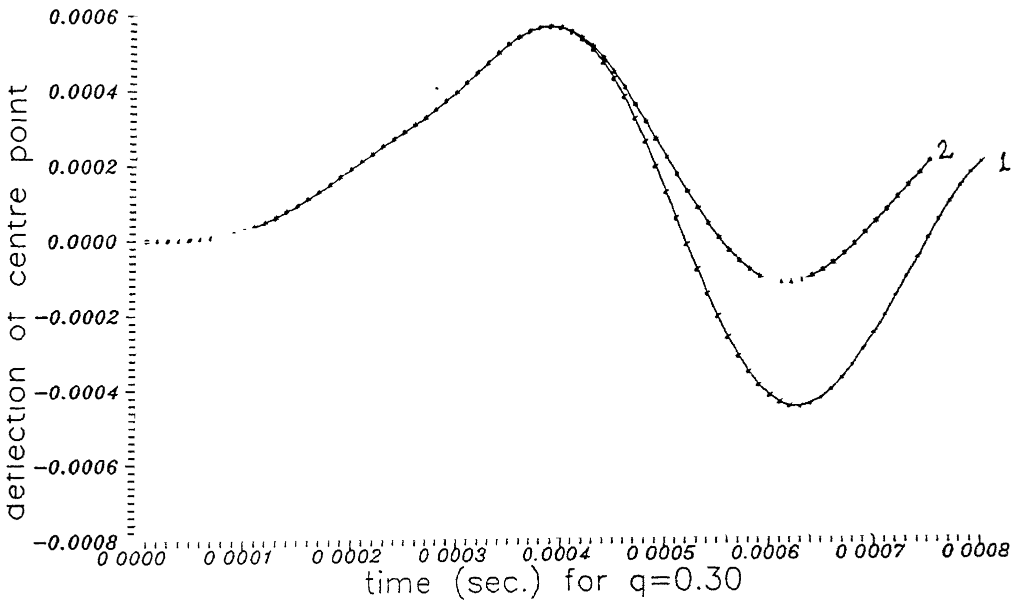
RT=0.0001

1- Purely elastic response
2- Elastoplastic response

Fig(4.8a): Effect of plastic behaviour on dynamic response



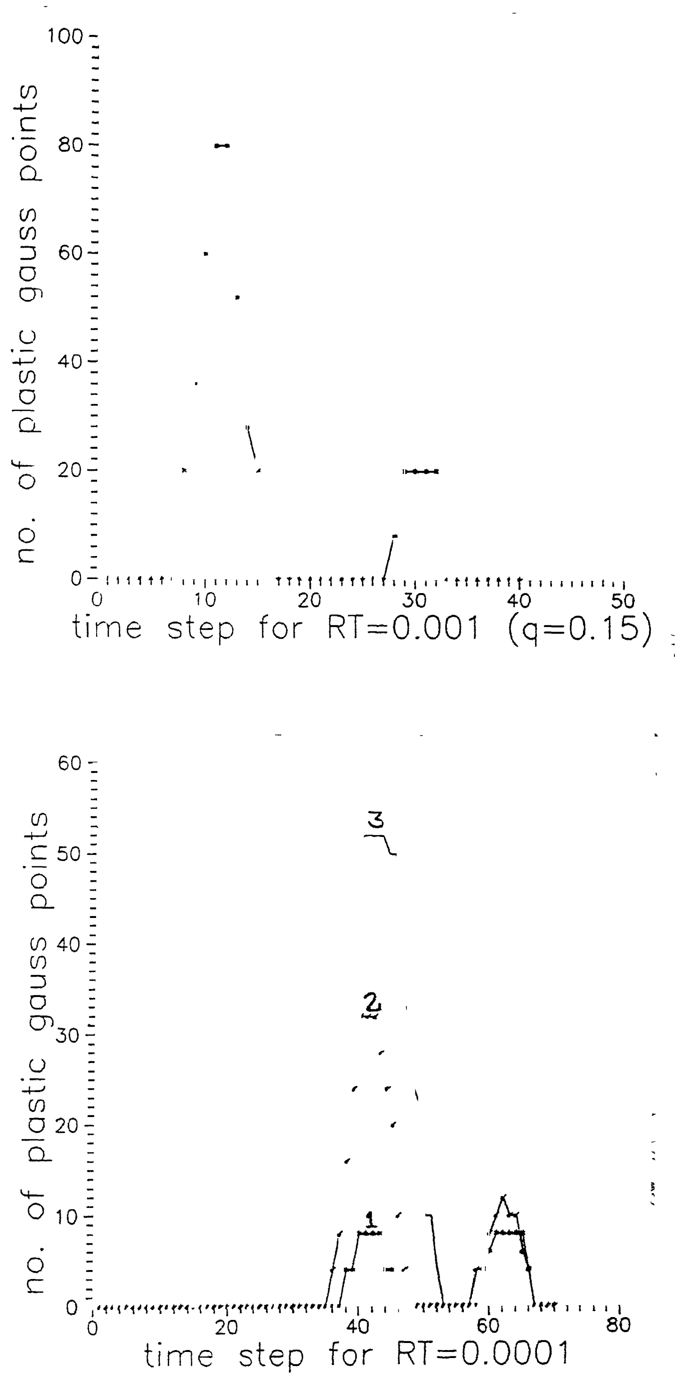
RT=0.0001



RT=0.0001

1- Purely elastic case
2- Elastoplastic case

Fig(4.8b): effect of plastic behaviour on dynamic response



1- q=0.20
2- q=0.25
3- q=0.30

Fig(4.9): Effect of strain hardening in loading and unloading. Peaks of the curves correspond to that of forcing function.

CHAPTER 5

CONCLUSIONS AND SCOPE FOR FUTURE WORK

5.1 Conclusions

Based on the present finite elastoplastic and dynamic analysis of a plate under dynamic and static loading, following conclusions can be verified,

1. In case of dynamic loading on the plate the plasticity is more than it was in purely static load with the same peak value.

2. The plasticity keeps on increasing as the rise time is decreased. And for a sufficient longer rise time there is no additional increase in the amount of plasticity over the static case.

3. As the plasticity is induced in the system, the energy is dissipated which acts as a dampener on the dynamic response and thus the oscillations reduces or vanishes.

4. The deflection during first peak load is more in elastoplastic case than purely elastic case. And in latter load peaks because of damping the deflection comes down for elstoplastic case.

5.2 Future Scope Of Work

The mesh can be made finer and the time step can be made smaller to observe the sharper impact loading and plasticity more accurately.

Several other loadings and different geometrical configurations can be tried. Also the cases of cyclic loading can be studied to see the permanent deflection after a given number of cycles.

Normally the small cracks are always present in the material, but that can not be modelled in the finite element. So to model the cracks we can make use of statistical method with varying yield stresses at different gauss points.

The effect of plasticity acts as an dampener in the dynamic response. The study can be useful where the system vibrations are to be avoided at the cost of permanent damage of some parts of the system.

For dynamic loading the time step can be selected in a optimum manner so that the computational time is less with considerable accuracy and stability.

REFERENCES

Ante, A. , 1987,'The explosive welding of metals.',First Int. Con. on Computational Plasticity, Barcelona.

Bathe, K. J. , 1990, Finite Element Procedures in Engineering Analysis, Prentice Hall of India Pvt. Ltd.

Borino, G., Caddemi, S. and Pollizzato, C., 1987,'Mathematical programming methods for evaluating dynamic deformations of elastoplastic structures.',First Int. Con. on Computational Plasticity, Barcelona.

Bazant, Z. P. , 1978, 'Spurios reflections of elastic waves in non uniform finite element grids. ', Comp. Meth. Appl. Mech. Engg, 16, 91-100.

Bazant, Z. P., Feng-Bao Lin and Pijaudier C. J., 1987,'Yield limit degradation: non local continuum model with local strain.', First Int. Con. on Computational Plasticity, Barcelona.

Bazant, Z. P. and Celep, Z., 1983, 'Spurious reflections of elastic waves due to gradually changing finite element size.' Int. J. Num. Meth. Engg., 19, 631-646.

Casadei, F., Halleux, J. P., Verzeletti, G. and Youstsos, 1987,' Application of numerical models to dynamic flow of steel specimens in the large test facility.', First Int. Con. on Computational Plasticity, Barcelona.

Dindore, U. S. , 1994, 'Dynamic effect of impact load on a plate with a crack: A finite element analysis', M. Tech. Thesis, I.I.T. Kanpur.

Giulio Maier and Giorgio Novati, 1987,'Deformation bounds for elastoplastic discrete structures with piecewise linear yield locus and non-linear hardening.',First Int. Con. on Computational Plasticity, Barcelona.

Hinton, E. and Owen, D.R.J., 1984, Software For Finite Element Methods in Plates and Shells., Pineridge Press, Swansea.

Hinton, E. and Owen, D.R.J., 1980, Finite Element in Plasticity: Theory and Practices., Pineridge Press, Swansea.

Kujawski, J., Miedzialowski and C., Ryzynski, W., 1987,'Dynamic analysis elastoplastic reinforced concrete wall systems.' First Int. Con. on Computational Plasticity, Barcelona.

Marques, J. M. M. C., 1987,'Code validation case studies in non-linear dynamic finite element analysis.',First Int. Con. on Computational Plasticity, Barcelona.

Mazza, G., Minasola, R., Molinaro, P. and Papa, A., 1987,'Dynamic

structural response of steel slab doors to blast loading.',First Int. Con. on Computational Plasticity, Barcelona.

Nayak, G. C., 1971,'Plasticity and large deformation problems by finite element method.', Ph. D. thesis, University of Wales, Swansea.

Nayak, G. C. and Zienkiewicz, O. C., 1972,'Elastoplastic stress analysis: Generalization for constitutive relations including strain softening.', Int. J. Num. Meth. Engg., 5, 113-35.

Owen, D. R. J. and Li, Z. H., 1987, 'Elastoplastic numerical analysis of anisotropic laminated plates by a refined finite element model.', First Int. Con. on Computational Plasticity, Barcelona.

Panzeca, T. and Polizzatto, C., 1987,'A finite element model for dynamic elastoplastic structural analysis.',First Int. Con. on Computational Plasticity, Barcelona.

Seron, F. J., 1990, , Finite element method for elastic wave propagation', Commun. In Appl. Num. Meth. ,6 ,359-368

Simo, J. C. and Taylor, R. L., 1985, 'Consistent tangent operators for rate dependent elastoplasticity.', Com. Meth. Appl. Mech. Engg., 48, 101-18

Timeshenko, S. and Woinowsky-Krieger, S., 1959, Theory of plates and shells, McGraw Hill Book Company Inc.

Yamada, Y., Yishimura, N. and Sakurai, T, 1968, 'Plastic stress strain matrix and its application for the solution of elastoplastic problems by finite element method.', Int. J. Mech. Sci., 10, 343-54.

Zienkiewicz, O. C., Qu, S., Taylor, R. L. and Nakazawa, S. , 1986, 'The patch test for mixed formulation.', Int. J. Numer. Meth., 23,1873-1887

Zienkiewicz, O. C. and Lefebvre, D.,1988, 'A robust triangular plate bending element of the Reissner-Mindlin type.', Int. J. Num. Meth., 26, 1167-1184

Zienkiewicz, O. C. and Taylor, R. L., 1991, The Finite Element Method, Fourth edition , vol. II, McGraw Hill Book Company.

FINITE ELEMENT ANALYSIS OF ELASTO PLASTIC DYNAMIC BEHAVIOUR OF PLATES UNDER IMPACT LOADING

A Thesis Submitted
in Partial Fulfillment of the requirement
For the Degree of
MASTER OF TECHNOLOGY

by
PRAVEEN PANDEY

to the
DEPARTMENT OF MECHANICAL ENGINEERING
INDIAN INSTITUTE OF TECHNOLOGY, KANPUR
FEBRUARY, 1996

CERTIFICATE

This is to certify that the present work titled '**Finite Element Analysis Of Elasto Plastic Dynamic Behaviour Of Plates Under Impact Loading**' by **Praveen Pandey** has been carried out under my supervision and has not been submitted elsewhere for the award of degree.

N N Kishore
12/2

Dr. N. N. KISHORE
Professor
Mechanical Engg. Dept.
I. I. T. Kanpur
INDIA

February, 1996

ACKNOWLEDGEMENTS

I express my deep sense of gratitude and indebtedness to Prof. N. N. Kishore for his invaluable guidance and constructive suggestions throughtout the present work.

I am grateful to my friends Rajesh Patnaik, S. K. Verma and Sanjeev Deshmukh for their help in one or other way.

It gives me immense pleasure to recall my association with Gagan, Raj, Sukku, Amit, Ramayan, Manoranjan, Sudipto, Madhukar, Amarendra and all other friends who made my stay at IITK a pleasant memory.

I wish to thank Prof. P. Kumar and his M.Tech. students for their co-operation in providing lab facilities at peak times.

And finally I pay my respects to All Mighty who provided me enough inner strength to carry on my work.

PRAVEEN PANDEY

February, 1996

Dedicated to

MY PARENTS

CONTENTS

NOTATIONS

A	Part of elemental stiffness matrix
AA	Slope of stress strain curve
{a}	Flow vector
a0-a7	Constant used in Newmark's Implicit scheme
B	Part of elemental stiffness matrix
BB	Matrix relating strains and displacements
C	A part of elemental stiffness matrix
C_1	Property matrix
c	Dilatational wave velocity
c2	Shear wave velocity
D	A part of elemental stiffness matrix
DD	Stress strain matrix
DD^{ep}	Elastoplastic stress strain matrix
E	A part of stress strain matrix
E	Modulous of elasticity
[F]	A part of elemental mass matrix
{f}	Load vector
G	Shear modulous of elasticity
h	Thickness
h_p	Elastic thickness
[K]	Stiffness matrix
$[K]^{ep}$	Elastoplastic stiffness matrix
L	Differential operator
L_w	Critical wave length
L_e	Effective length of an element
[M]	Mass matrix
{M}	Moment vector
N	Shape function

P	A part of elemental stiffness matrix
q	Uniformly distributed load
S	Shear force
T	Edge moment
$\{U\}$	Displacement vector
u	Displacement in X-direction
v	Displacement in Y-direction
w	Displacement in Z-direction
δ, α	Constants used in Newmark's Implicit scheme
ϵ	Strain
Γ	Boundary
γ	Shear strain
ν	Poisson's ratio
σ	Stress
θ	Rotation
Ω	Area
υ	Differential operator
Δt	Time step

ABSTRACT

In practical cases when a plate is subjected to a dynamic loading , the geometric and material linearity may not be preserved. This study is aimed to investigate the effect of plasticity in the dynamic response . As there are no closed form solutions available owing to the complexity of the problem ,numerical methods are used to solve the problem.

For this computational work an efficient FEM code is developed using the "robust triangular element" developed by Zienkiewiz [1991] . "Mindlin plate theory" in the form of mixed formulation is used for modelling plate bending. The inertia effect of the element is represented by consistent mass matrix.

As the stresses are varying along the thickness, the plasticity should be applied together with elasticity for every iteration. This introduces the concept of "elastic height", which decides the stresses in the plate. And to find out the elastoplastic stiffness matrices, the integration should be done in both elastic core and plastic outer region.

The computer code is first used to solve a standard problem and after validation it was used for dynamic cases of impact loading. The dynamic loading increases the plasticity and in turn the plasticity in the material acts as an dampener to the dynamic oscillatory response. The interdependence of plastic and dynamic behaviours is analyzed in the present work.

1.1 GENERAL INTRODUCTION

Plasticity in the plate is different from a simple two dimension analysis of the plasticity. The main difference is the variation of stress along the thickness. As the first approximation of the plate stress and strain in the thickness direction are neglected [Reissner Mindlin plate formulation]. And if the plate is thin then the out of plane stresses (τ_{xz}, τ_{yz}) are also neglected. For a thick plate in-plane-stresses($\sigma_x, \sigma_y, \tau_{xy}$) varies with the height while out of plane stresses remain almost constant along the height. This variation of the stresses affects the size of the elastic region in the core of the plate.

In general plate formulation is made in terms of lateral forces and moments. In plastic case since the in-plane stresses are different in the elastic core and outer plastic zone, the calculation of the stress resultant moments becomes a function of the size of elastic core. The whole thickness is not plastic, proper weightages are to be given to the elastic and plastic stiffness matrices to calculate effective stiffness matrix. Calculations of unbalances after the integration of dynamic analysis and the plastic analysis is to be done to incorporate both the aspects properly. These taken together increases the complexity of the problem.

1.2 LITERATURE SURVEY

The thin plate theory developed by Kirchhoff is modified for thick plates by Reissner and Mindlin by relaxing the assumption that the normals to the midplane remain normal. This was done since the shear force terms can not be neglected in many loading and support conditions. Reissner and Mindlin theories can be

extended to thick plates and thus known as Reissner-Mindlin formulation. Regarding Finite Element Method a difference between thin plate and thick plate analysis is that in former it is possible to represent the state of deformation by one quantity deflection w and thus C^1 continuity of w is needed, while in latter w can have C^0 continuity along with rotation and shear degree of freedom.

While analytical solutions remain difficult for thick plates, the advent of Finite Element Method made thick plate theory simpler to implement. Thick plate theory on the imposition of additional assumptions behaves as thin plate theory. It is more natural to start with thick plate formulation and make transition to thin plate theory by imposition of certain relation between degrees of freedom [Zienkiewicz and Taylor,1991]. In thick plate formulation instead of one approximate function for w (for calculation of rotational angle), independent interpolation functions for deflection w and rotation θ are taken. During formulation, if the shear forces are eliminated, the formulation is known as "irreducible formulation" and when functions for shear forces are also retained the formulation is known as "hybrid formulation". The latter formulation gives more degree of freedom and so it is preferred to work with in most of the cases [Zienkiewicz and Taylor,1991]

Another aspect is the selection of the element to perform satisfactorily for thick plates as well as thin plates. Many elements are attempted to avoid the problems of locking and singularity but the results of "triangular robust element" are found favourable in this regard [Zienkiewicz and Lefebvre,1988].

For determination of plasticity three things are required: an yield criteria, flow rule and plastic stress strain law. A

fundamental difference between plastic and elastic analysis is that in elastic solution total stress can be evaluated from the total strain alone, while in the plastic response evaluation of stress strain at any time depends on the stress strain history. This depends on the experimental results which are termed as yield criteria. Commonly used yield criterion are continuous one but piecewise linear yield criterion can also be used [Giulio Maier et al.,1987].

Yield criterion defines the onset of plasticity and flow rule governs the relationship between stress and plastic strain increment after the yield point. Flow rule requires the definition of a potential which is taken to be the same as yield surface for the case of associated plasticity. The plastic stress strain law has been defined for nonlinear, linear and perfectly plastic cases [Yamada et al., 1968]. In latter two cases the total stress strain curve is bilinear and in former case usually some power law satisfying the experimental results is defined. Flow rule, thus can be used in plastic cases by forming a plastic potential and then defining the plastic strains by the same. When the plastic potential surface is the same as the yield surface, the plasticity is termed as "associated" otherwise the plasticity remains "unassociated". The flow rules for loading and unloading are not same, and that makes the problem difficult.

Out of many iterative schemes to solve the above non-linear problem, Newton-Raphson method is found favourable in terms of ease in mathematical formulation and in computational effort [Simo and Taylor, 1986]. In Newton-Raphson method, incremental strain is multiplied to plastic stress strain matrix to get the stresses, while in Runge-Kutta method the incremental strain is subdivided in many subincrements then individual subincremented

strains are multiplied to their corresponding stress strain matrices to get subincremented stress. These stresses are added up to get incremental stress. This is known as "subincrementation" [Nayak,1971; Bathe,1990].

For plasticity in the Mindlin plates, it is suggested to use full three dimensional analysis of stress field. Inclusion of shear terms in the yield function allows the spread of plasticity from the extreme fibre to entire plate thickness [Hinton and Owen,1980]. A very thick plate can be divided in many layers of thin plates and with the Mindlin plate theory,the problem is solved [Hinton and Owen,1980]. But the triangular robust element is proved favourable for transition from thick to thin plates, and the non layered approach can be used for moderately thick plates [Zienkiewicz and Lefebvre,1988]. Elastoplastic numerical experiments has been carried out adopting a full three dimensional version and different criteria of yield in the past [Hinton and Owen,1984].These results show excellent agreement in static elastic analysis and the prediction of vibrational frequencies and buckling loads. These results forms the basis of comparison for the present work.

In elastodynamic, a detailed analyses of various time integration schemes from point of view of accuracy and speed, were reported by [Seron et al.,1990; Zienkiewicz and Taylor,1991; Bathe,1990]. It is concluded that,even though central difference scheme is the best in terms of computational speed and cost, Newmark's constant average scheme gives better accuracy and offers unconditional stability. Whereas, the time step size has negligible effects when central difference scheme is employed [Bazant and Celep,1983].

In elastoplastic dynamic analysis based on Mindlin plate, the work started quite late. Most of the work is in the area of concrete structure under dynamic loading [Kujawski et al.,1984].

Experimental information on material behaviour is scarce for most structural materials [Hinton and Owen,1980]. Instantaneous yield stress is significantly influenced by the rate of straining, also the value of elasticity modulus is found to be dependent on the strain rate for structural material those varies from the materials of limited ductility to brittle elastic behaviours. For this purpose a better understanding of observed phenomenon and underlying microscopic behaviour is required [Hinton and Owen, 1980].

In loading and unloading the yield point of the material changes because of strain hardening. A model for yield limit degradation has been developed by Bazant [1987]. Other study [Panzeca et al.,1987] deals with a transformation matrix that takes care of yield point variation according to the loading and history.

Application of numerical model to dynamic flow of steel specimen has been done by Casadei et al.[1987]. The results compared favourably with the experimental result. For beams and rods a code validation case study is done by Marques [1987]. An elasto-plastic one degree of freedom numerical model was presented for steel slab under blast loading [Mazza et al.,1987]. The results were compared with static elastoplastic loading. Also both of the results were compared with experimental data. It was found out that in the dynamic loading conditions the displacements were intensified at the plastic zone compared with those of static loading. In the area of explosive welding also

the study was carried out [Ante, 1987]. In this case welding of two concentric tubes were considered while explosive was put inside the inner tube. The specimen experienced both mechanical as well as thermal shock. In this problem plasticity was induced by thermal load also.

PRESENT WORK

The present work deals with the study of the effect of plasticity in plates under various type of loadings both static and dynamic.

A mixed finite element analysis using Reissner Mindlin plate theory is used to model the plate bending. To avoid locking and singularity a triangular robust element (T6/3 B3) [Zienkiewicz and Taylor, 1991] is used.

Chapter 2 deals with the theoretical part of the formulation of plate bending and plasticity. Chapter 3 deals with the finite element implementation of the plastic analysis and its integration with dynamic analysis. Chapter 4 describes the validation of the code and the behaviour of plasticity in dynamic loading and effect of plasticity on the dynamic response. Chapter 5 presents the conclusions and the suggestions for future work.

CHAPTER 2

BASIC EQUATIONS OF ELASTOPLASTIC DYNAMIC PLATE ANALYSIS

2.1 INTRODUCTION

This chapter presents the basic equations governing the plate under elastoplastic and dynamic conditions. As explained in earlier chapter, by appropriate element selection and formulation Mindlin plate theory can be used for thick and thin plates.

The Mindlin plate theory makes the following assumptions,

1.The strain and the stress components in Z-direction are negligible (σ_z and $\epsilon_z \approx 0$) (fig 2.1), where Z-direction represents the thickness direction.
2.The normals to the middle plane of the plate remains straight during deformation.

Applying above assumptions, static equilibrium equations for plate bending problem can be obtained [Timoshenko,1959] in terms of midplane rotations(θ_x,θ_y),lateral displacements(w) of the midplane and the shear forces(S_x,S_y) at the cross-section. Sign convention and the variables are shown in fig(2.1).

Stress resultant moments and shear forces are defined in terms of stress variables as follows:

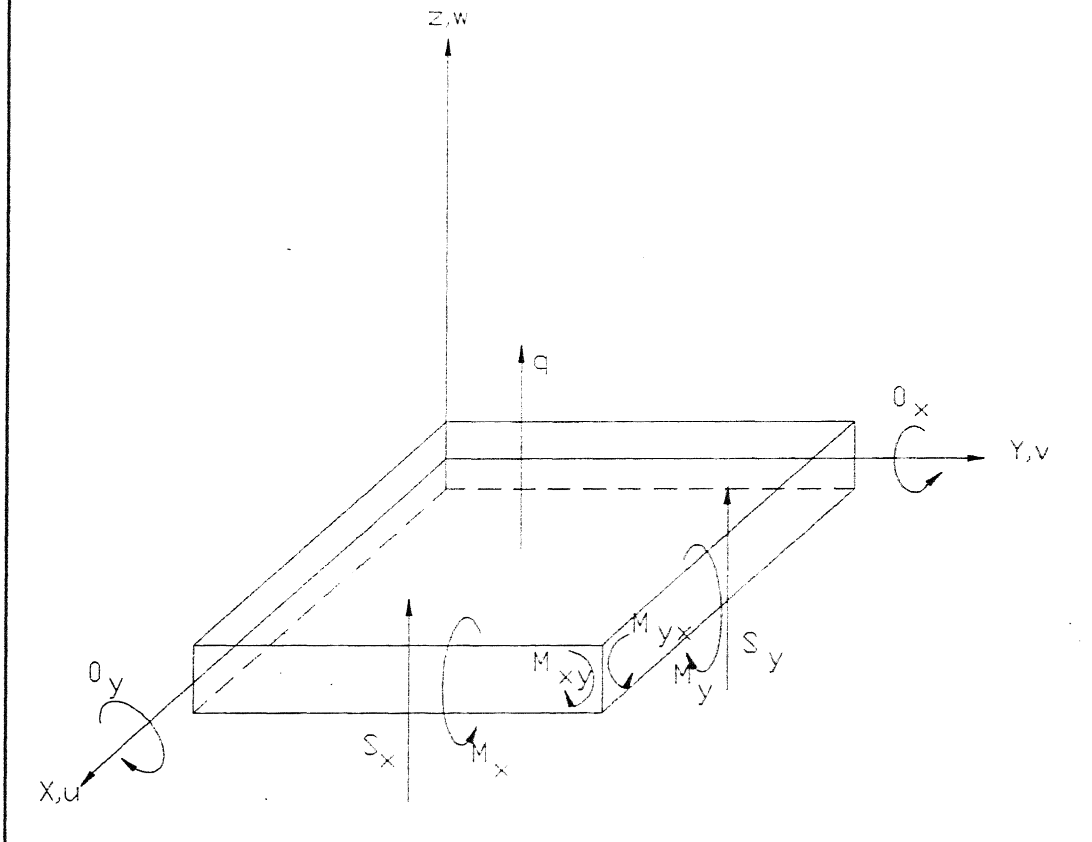


FIG 2.1

$$M_x = \int_{-h/2}^{h/2} \sigma_x \, z \, dz \quad , \quad M_y = \int_{-h/2}^{h/2} \sigma_y \, z \, dz \tag{2.1a}$$

$$M_{xy} = \int_{-h/2}^{h/2} \tau_{xy} \, z \, dz \tag{2.1b}$$

$$S_x = \int_{-h/2}^{h/2} \tau_{xz} \, dz \quad , \quad S_y = \int_{-h/2}^{h/2} \tau_{yz} \, dz \tag{2.1c}$$

For the dynamic analysis inertia forces due to mass and moment of inertia are considered in equilibrium equations. Formulating equilibrium equation using d' Alembert's principle, the following equations are obtained.

$$\frac{\partial M_x}{\partial x} + \frac{\partial M_{xy}}{\partial y} - S_x - \rho \frac{h^3}{12} \ddot{\theta}_x = 0 \tag{2.2a}$$

$$\frac{\partial M_{xy}}{\partial x} + \frac{\partial M_y}{\partial y} - S_y - \rho \frac{h^3}{12} \ddot{\theta}_y = 0 \tag{2.2b}$$

$$\frac{\partial S_x}{\partial x} + \frac{\partial S_y}{\partial y} + q - \rho h \ddot{w} = 0 \tag{2.2c}$$

Strains at a height of z from the centre plane are given as,

$$\epsilon_x = z\frac{\partial\theta_x}{\partial x} \quad , \quad \epsilon_y = z\frac{\partial\theta_y}{\partial y} \tag{2.3a}$$

$$\gamma_{xy} = z\left[\frac{\partial\theta_x}{\partial y} + \frac{\partial\theta_y}{\partial x}\right] \tag{2.3b}$$

$$\gamma_{xz} = \theta_x + \frac{\partial w}{\partial x} \quad , \quad \gamma_{yz} = \theta_y + \frac{\partial w}{\partial y} \tag{2.3c}$$

2.2 THE MIXED FINITE ELEMENT FORMULATION

The finite element formulation involves three sets of variables θ, S, w. These variables at any point within the element are expressed in terms of nodal parameters (θ_x, θ_y, S_x, S_y, w) of the element using shape functions.

$$\theta = \begin{bmatrix}\theta_x\\ \theta_y\end{bmatrix} = N_\theta\overline{\theta} \tag{2.4a}$$

$$S = \begin{bmatrix}S_x\\ S_y\end{bmatrix} = N_s\overline{S} \tag{2.4b}$$

$$w = N_w\overline{w} \tag{2.4c}$$

Where θ, S, w are the nodal values of respective variables.

Now using eq(2.3) and eq(2.1) the eq(2.2) can be rewritten as follows.

$$L^T D L \theta - S - \rho \frac{h^3}{12} \ddot{\theta} = 0 \qquad (2.5a)$$

$$\theta - C_1 S + \nabla w = 0 \qquad (2.5b)$$

$$\nabla^T S + q - \rho h \ddot{w} = 0 \qquad (2.5c)$$

Where L, ∇ operators and C_1 matrices are given by,

$$L = \begin{bmatrix} \frac{\partial}{\partial x} & 0 \\ 0 & \frac{\partial}{\partial y} \\ \frac{\partial}{\partial y} & \frac{\partial}{\partial x} \end{bmatrix} \quad , \quad \nabla = \begin{bmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \end{bmatrix} \quad , \quad C_1 = \frac{1}{Gh} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$D = \frac{Eh^3}{12(1-\nu^2)} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix}$$

Now by Galerkin method, using eq(2.5) and eq(2.4), and then integrating over the area of the element, the following equations are obtained.

$$A\overline{\theta} + B\overline{S} + E\overline{\theta} = f_\theta \qquad (2.6a)$$

$$B^T\overline{\theta} - P\overline{S} + C\overline{w} = 0 \qquad (2.6b)$$

$$C^T\overline{S} + F\overline{W} = f_w \tag{2.6c}$$

Where,

$$A = \int_\Omega (LN_\theta)^T D(LN_\theta)\, d\Omega \quad , \quad B = \int_\Omega N_\theta^T N_s d\Omega$$

$$C = \int_\Omega N_s^T (\nabla N_w)\, d\Omega \quad , \quad P = \int_\Omega N_s^T C_1 N_s d\Omega$$

$$f_\theta = \int_{\Gamma_t} N_\theta^T\ T\ d\Gamma \quad , \quad f_w = \int_\Omega N_w^T\ q\ d\Omega$$

$$E = \frac{\rho\ h^3}{12}\int_\Omega N_\theta^T\ N_\theta\ d\Omega \quad , \quad F = \rho\ h\int_\Omega N_w^T\ N_w\ d\Omega$$

In the above equations the vector T represents two moment components imposed on the boundary by the traction and q is the intensity of the imposed lateral force. The shape functions N_θ, N_w are chosen to have C_0 continuity, but because of shear forces are defined only for inside nodes of the element, N_s can be discontinuous between the elements. Such a choice allows S to be defined locally for each element and thus be eliminated when P is a non-singular matrix. Thus the problem includes only θ and w as variables.

The final formulation in terms of θ and w, after condensing S, becomes :

$$\begin{bmatrix} A + BP^{-1}B^{T} & BP^{-1}C \\ C^{T}P^{-1}B^{T} & C^{T}P^{-1}C \end{bmatrix}\begin{bmatrix} \theta \\ \overline{w} \end{bmatrix} + \begin{bmatrix} E & 0 \\ 0 & F \end{bmatrix}\begin{bmatrix} \theta \\ \overline{w} \end{bmatrix} = \begin{bmatrix} f_{\theta} \\ f_{w} \end{bmatrix} \tag{2.7}$$

In short eq(2.7) can be written as,

$$[K]^{e}\{u\} + [M]^{e}\{\ddot{u}\} = \{F\}^{e} \tag{2.8}$$

2.3 PLASTIC ANALYSIS

When the stresses go beyond the yield value, then stiffness matrix changes depending on the individual stress component, effective stress and flow rule. In such cases no direct solution can be used, so iterative schemes are used to solve the problem.

To do the non-linear problems, the following steps are used:

a. Calculation of stress and strain

After obtaining the maximum possible elastic solution, the correspondig strains are calculated by the following formula which is obtained by substituting eq(2.3) using the variables from eq(2.4).

$$\{\epsilon\} = \begin{bmatrix} z\,(LN_{\theta}) & 0 \\ N_{\theta} & \nabla N_{w} \end{bmatrix}\{u\} = [BB]\,\{u\} \tag{2.9}$$

In a plate problem inplane strains($\epsilon_x, \epsilon_y, \gamma_{xy}$) and thus

stresses($\sigma_x, \sigma_y, \tau_{xy}$) are the functions of the height, while out of plane strains(γ_{xz}, γ_{yz}) and stresses(τ_{xz}, τ_{yz}) are constant across the thickness. And the stress strain relationship,

$$\{\sigma\} = [DD]\{\epsilon\} \tag{2.10a}$$

$$[DD] = \frac{E}{1-\nu^2}\begin{bmatrix} 1 & \nu & 0 & 0 & 0 \\ \nu & 1 & 0 & 0 & 0 \\ 0 & 0 & \frac{1-\nu}{2} & 0 & 0 \\ 0 & 0 & 0 & \frac{1-\nu}{2k} & 0 \\ 0 & 0 & 0 & 0 & \frac{1-\nu}{2k} \end{bmatrix} \tag{2.10b}$$

Where

$$\{\sigma\} = \begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \\ \tau_{xz} \\ \tau_{yz} \end{bmatrix} \quad , \quad \{\epsilon\} = \begin{bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \\ \gamma_{xz} \\ \gamma_{yz} \end{bmatrix}$$

In eq(2.10b) k=1.2 is a factor for improving the performance of the analysis, and this accounts for non uniform shear stress distribution. The stress and strain vectors have five above mentioned components.

b. Yield Criteria

In the present analysis Prandtl-Reuss relation is made use to determine the onset of yield.

$$\sqrt{\sigma_x^2 + \sigma_y^2 - \sigma_x\sigma_y + 3\tau_{xy}^2 + 3\tau_{xz}^2 + 3\tau_{yz}^2} = \sigma_{ys} \tag{2.11}$$

c. Fraction Of Plasticity

As the load is increased, it is known, that the outer most fibres become plastic first and the plasticity spreads in to the core and across the plate. At the same time whole thickness is not plastic. So to find above what height the plate goes plastic, the height of elasticity is calculated as(fig 2.1),

$$h_p = \left[\frac{\sigma_{yield}^2 - 3(\tau_{xz}^2 + \tau_{yz}^2)}{\sigma_x^2 + \sigma_y^2 - \sigma_x\sigma_y + 3\tau_{xy}^2}\right]^{1/2} \frac{h}{2} \tag{2.12}$$

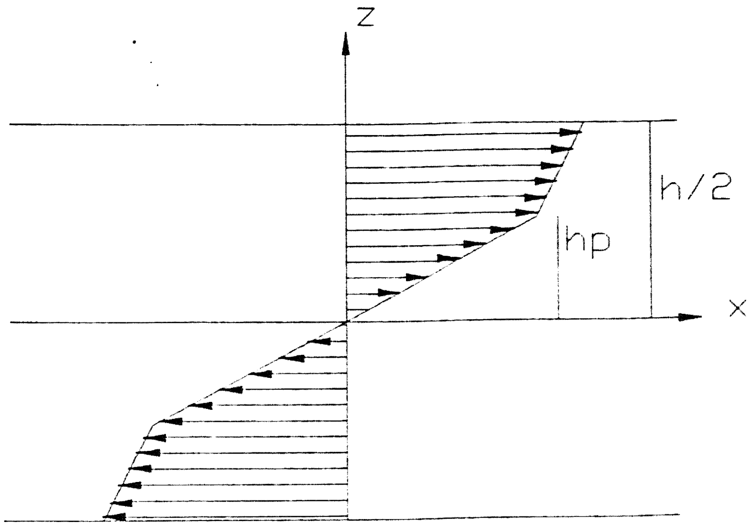
If the numerator of the term inside the root sign goes negative, the whole thickness goes plastic.

d. Flow Rule

Plastic materials are governed by flow rule between stress and plastic strain increment. The flow rule is obtained by the normality principle. The relation between effective stress and strain in simple terms can be understood by the power law in one direction. The power law is an experimental agreement of the material properties in the plastic range.

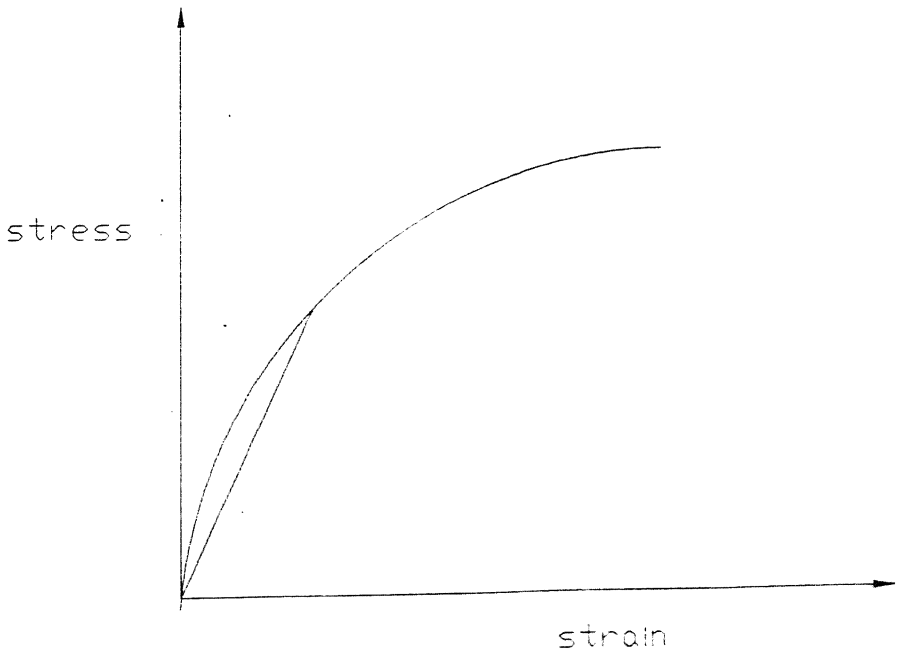
$$\sigma = K\epsilon^n \tag{2.13}$$

Where K and n are material constant and calculated experimentally. The eq(2.13) governs the relation only after yield point. The power law is shown in fig(2.3).



VARIATION OF STRESS ALONG HEIGHT

FIG(2.2)



FIG(2.3)

e. Plastic Stress-Strain Matrix($[DD]^{ep}$)

If a point in the plate goes plastic then the plastic stress strain matrix is used at that point to calculate the incremental stresses from incremental strains in the plastic region.

$$\{\delta\sigma\} = [DD]^{ep}\{\delta\epsilon\} \tag{2.14}$$

Where

$$[DD]^{ep} = [DD] - \frac{[DD]\{a\}\{a\}^T[DD]}{AA + \{a\}^T[DD]\{a\}}$$

Here [DD] is the property matrix, AA is the slope of stress-strain curve governed by elastoplstic strain stress relation and {a} is the flow vector which depends on the plasticity criteria.

f. Iterative scheme

The unbalance forces to be calculated by commonly used equation,

$$\{f\} = \int [BB]^T\{\sigma\}\,dV \tag{2.15}$$

does not work in this mixed formulation because of the following reasons.

1. The eq(2.15) can be used if the following expression satisfies the stiffness matrix of an element.

$$[K]^e = \int [BB]^T [DD] [BB] \, dV \tag{2.16}$$

2. Also if the above equation is assembled to form global stiffness matrix for this particular robust element, the global stiffness matrix goes singular.

Answer to these difficulties is found in the mixed formulation which incorporates shear forces also as degree of freedom. Inclusion of such degree of freedom avoids the defects of singularity and locking. Presence of N_s changes the character of stiffness matrix, while the same is not present in eq(2.16).The eq(2.7) is derived from "mixed formulation" while eq(2.15) and eq(2.16) represent the "irreducible formulation".

So leaving eq(2.15) and eq(2.16) aside, another iterative scheme is tried which make use of original elemental stiffness matrix.

For an element, if it goes plastic at a given load then the incremental deformations should satisfy the following equation.

$$[K]^{ep}\{\delta u\} = \{\delta f\} \tag{2.17}$$

Here $[K]^{ep}$ is the elastoplastic stiffness matrix. Since this matrix is a function of $[DD]^{ep}$ matrix and thus a function of stresses, the global [K] matrix needs updating. So to avoid such recalculations use of the iterative scheme of eq(2.17) is implemented in the following form,

$$[K]^e\{\delta u\} = \{\delta f\} + [[K]^e - [K]^{ep}]\{\delta u\}$$

Now displacement vector on left hand side is treated as displacement vector obtained after nth iteration while the displacement vector on right hand side is the vector before the iteration and is used to calculate unbalance forces. Now for showing the iterative scheme the eq(2.18) can be rewritten as,

$$[K]^e\{\delta u\}_n = \{\delta f\} + [[K]^e - [K]^{ep}]\{\delta u\}_{n-1} \qquad (2.18)$$

The geometrical representation of eq(2.18b) is shown in the fig(2.4).

g. Convergence Criterion

The "error" in each iteration is represented as follows,

$$e = \{u_n\} - \{u_{n-1}\}$$

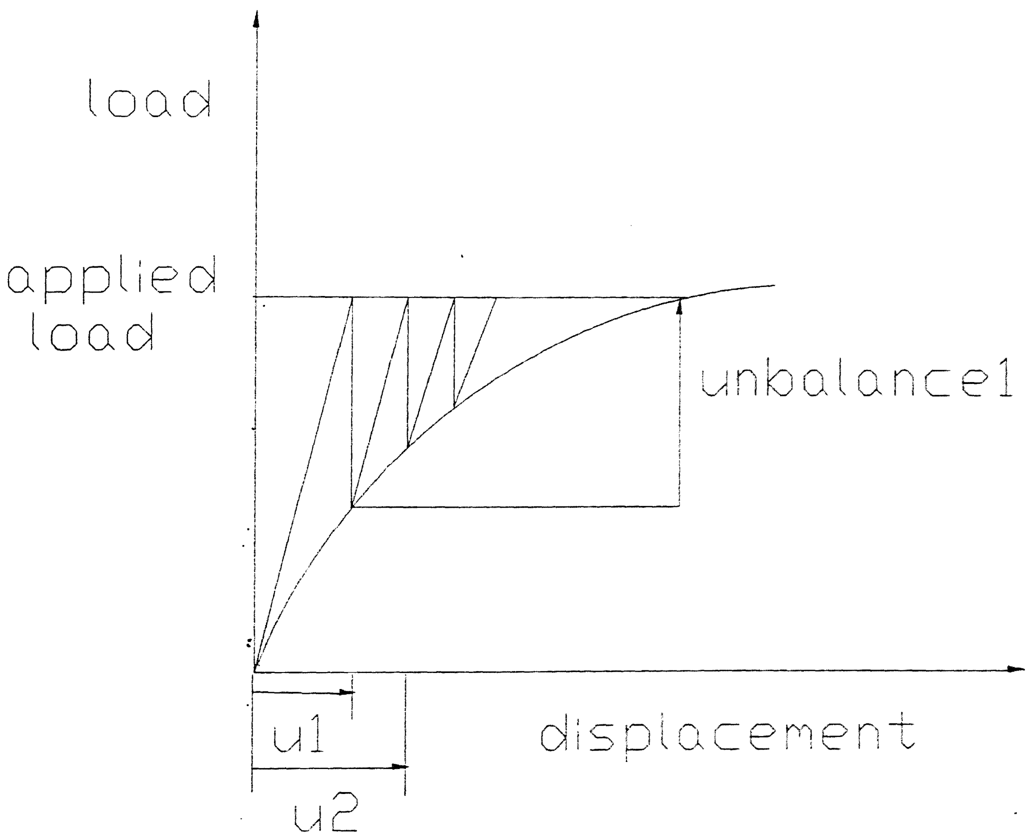
The norm of this error vector will be,

$$|e| = [e^T e]^{\frac{1}{2}}$$

And convergence is said to be achieved when,

$$\frac{|e|}{|u_n|} \leq \alpha \qquad (2.19)$$

Where α is the convergence limit.



FIG(2.4)

h. Effective Plastic Stiffness $[K]^{ep}$ Matrix

$[K]^{ep}$ matrix is the plastic stiffness matrix for an element. Since the whole plate may not go plastic and some portion may still remain elastic, the calculation of this matrix have to made carefully.

Eq(2.14) shows incremental stress strain $[DD]^{ep}$ matrix for full plasticity and eq(2.10b) shows stress strain matrix [DD] for full elasticity. In $[K]^{e}$ matrix eqn(2.8) the material properties are used at two places in form of [D] and $[C_1]$ matrices. Now matrices [DD] and $[DD]^{ep}$ are defined in the following way.

$$[DD] = \begin{bmatrix} [DD_1] & 0 \\ 0 & [DD_2] \end{bmatrix} \quad , \quad [DD]^{ep} = \begin{bmatrix} [DD_1]^{ep} & 0 \\ 0 & [DD_2]^{ep} \end{bmatrix}$$

Where $[DD_1]$, $[DD_1]^{ep}$ are 3x3 and $[DD_2]$, $[DD_2]^{ep}$ are 2x2 matrices. Now effective plastic stress strain matrix can be written in the following way.

$$[D]^{ep} = \sum_{j=1}^{7}\left[\frac{2h^3}{3}[DD_1] + \left(\frac{h^3}{12} - \frac{2h_p^3}{3}\right)[DD_1]^{ep}\right]w_j \quad (2.20)$$

Which represents the gauss numerical integration with 7 gauss points of the element. $[DD_1]^{ep}$ and h_p are also different for different gauss points.

And

$$[C_1]^{ep-1} = \sum_{j=1}^{7}\left[2h_p[DD_2] + (h-2h_p)\,[DD_2]^{ep}\right]w_j \quad (2.21)$$

Fitting $[D]^{ep}$ and $[C_1]^{ep}$ in place of $[D]$ and $[C_1]$ in the eq(2.6), The $[K]^{ep}$ matrix in place of $[K]^{e}$ matrix is obtained.

The loading and unloading parts change the characteristic of stress strain curve and the yield stress. This aspect is also included and discussed in next Chapter with other aspects in implementation of the computer code.

CHAPTER 3

IMPLEMENTATION OF FINITE ELEMENT CODE

This chapter explains the salient aspects of numerical method in developing the FEM code, including element selection.

3.1 Element Selection

Triangular elements are easier for discretizing complex geometries. In literature for different types of problems different type of elements are suggested as they should be free from defects of locking and singularity. Also the performance of the element under different type of loadings and boundary conditions should be stable. "Robust triangular element" with suitable nodal variables satisfy the stability conditions.

The choice of the shape functions and variables should be such that the Babuska-Brezzi [Zienkiewicz and Lefebvre, 1988] conditions are satisfied by the system and thus, stability is ensured. The most vital and strictly necessary condition for the non-singularity of the system given by eqn(2.8) is that,

$$\alpha = \frac{n_\theta + n_w}{n_s} \geq 1 \tag{3.1a}$$

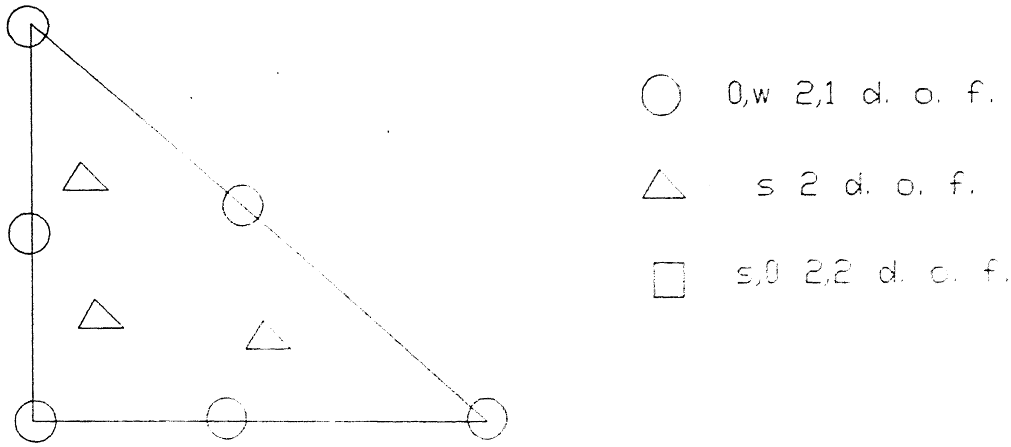
$$\beta = \frac{n_s}{n_w} \geq 1 \qquad (3.1b)$$

Where n_θ, n_s and n_w stand for the number of variables for rotation, shear force and lateral displacements respectively in an element. If either of the two conditions are violated, the locking behaviour will occur in the numerical solution.

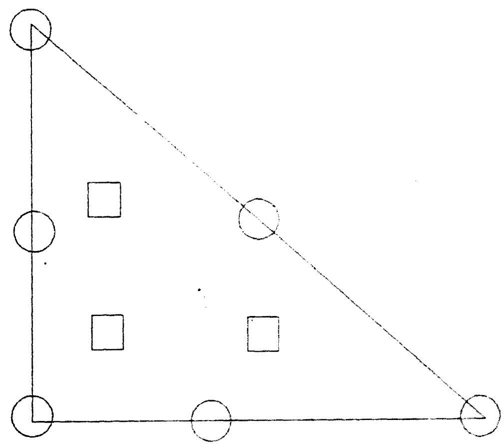
For the dynamic analysis if an implicit time integration scheme is used, it is necessary to use higher order finite elements and a consistent mass matrix. The higher order elements are effective in the representation of bending behaviour. These elements should be employed with consistent load vector, in that the midside and corner nodes are subjected to their appropriate load contribution in the analysis.

In the mixed finite element formulation, triangular element (T6/3) has six boundary nodes which are used to determine a quadratic variation of θ and w [Zienkiewicz and Lefebvre, 1988]. Shear forces are determined by a linear field defined by three internal nodes.

When standard patch tests [Zienkiewicz and Lefebvre,1988;Zienkiewicz et al.,1986] are used with this element, the results indicate that, the element has incipient locking possibility and thus it is not a satisfactory element. But the examination of results indicate that the satisfaction of the requirements can be achieved by the addition of the θ variables in the interior of the element. The three internal



(a) T6/3 (24 d. o. f.)



(b) T6/3 B3 (30 d. o. f.)

FIG(3.1)

nodes created have the shape functions used of the form $L_1^2L_2L_3$, provide a new element (T6/3 B3) which passed the same patch test successfully in all cases [Zienkiewicz and Lefebvre, 1988].

This particular element is chosen for the present analysis. It has 30 degree of freedom, and after the condensation of shear force terms the number of variables reduces to 24.

3.2 Time Integration scheme

The eqn(2.8) is integrated by a numerical procedure. The integration is based on two ideas.

1. Static equilibrium including inertia force is achieved at discrete time intervals Δt.

$$[K]\,^{t+\Delta t}\{u\} + [M]\,^{t+\Delta t}\{\ddot{u}\} = {}^{t+\Delta t}\{F\} \tag{3.2}$$

2. Variation of displacement, velocity and acceleration within each time step is assumed. This assumption decides the accuracy and stability of the solution procedure.

The idea no. 2 decides the time integration scheme. The choice of a scheme depends on the finite element idealization. However, the finite element idealization to be chosen depends in turn on the actual wave velocity of the medium to be analyzed. It follows, therefore the selection of an appropriate finite element idealization of a problem and the choice of effective integration scheme for the response solution are closely related and must be considered together.

Out of many schemes "Newmark's implicit scheme, is most

accurate and unconditionally stable.

3.2a Newmark Implicit Scheme

The Newmark's constant acceleration scheme is written as follows [Bathe, 1990],

$$^{t+\Delta t}\dot{U} = {}^{t}\dot{U} + [(1-\delta)\ {}^{t}\ddot{U} + \delta\, {}^{t+\Delta t}\ddot{U}]\,\Delta t \tag{3.3a}$$

$$^{t+\Delta t}U = {}^{t}U + {}^{t}\dot{U}t + [(\frac{1}{2}-\alpha)\,{}^{t}\ddot{U} + \alpha\, {}^{t+\Delta t}\ddot{U}]\,\Delta t^{2} \tag{3.3b}$$

Where α and δ are parameters that control integration accuracy and stability. Newmark originally proposed an unconditional stable scheme, in which case $\delta = 0.5$ and $\alpha = 0.25$.

Using eqn(3.2) and eqn(3.3) final form can be obtained as,

$$[K+a_0M]\,{}^{t+\Delta t}U = {}^{t+\Delta t}\{F\} + [M]\,(a_0{}^{t}U + a_2{}^{t}\dot{U} + a_3{}^{t}\ddot{U}) \tag{3.4a}$$

$$^{t+\Delta t}\ddot{U} = a_0\,({}^{t+\Delta t}U - {}^{t}U) - a_2{}^{t}\dot{U} - a_3{}^{t}\ddot{U} \tag{3.4b}$$

$$^{t+\Delta t}\dot{U} = {}^{t}\dot{U} + a_6{}^{t}\ddot{U} + a_7{}^{t+\Delta t}\ddot{U} \tag{3.4c}$$

Where,

$$a_0 = \frac{4}{\Delta t^2} \quad , \quad a_2 = \frac{4}{\Delta t} \quad , \quad a_3 = 1$$

$$a_6 = \frac{\Delta t}{2} \quad , \quad a_7 = \frac{\Delta t}{2}$$

The implicit scheme, as can be seen from the final formulation requires inversion of $[K+a_0M]$ matrix. For this purpose Cholesky decomposition technique is computationally efficient.

3.2b Time step

After deciding on time integration scheme, it is necessary to use a time step suitable with finite element mesh. If the critical wavelength in the medium is denoted by L_w, then the time step can be decided as,

$$\Delta t = \frac{L_w}{c\, n}$$

And the effective length of a finite element should be

$$L_e = c\, \Delta t$$

Where c is wave speed and 'n' is the number of nodes per critical wavelength.

The effective wavelength and corresponding time step must be able to represent the complete bending wave accurately and is chosen appropriately depending on the kind of element idealization and time integration scheme used. It is reported that at least 8 nodes per shortest wavelength are required in order to produce all artifacts of wave propagation [Seron,1990].

However, for higher order (parabolic and cubic) continuum elements the time step has to be further reduced, because the interior nodes are stiffer than the corner nodes [Bathe ,1990]. In addition for T6/3 B3 ,the internal θ variables are quartic functions. Also in the plate elements the flexural modes also affect the time step. In the actual computation appropriate time step is to be selected.

3.3 Implementation Of Plastic And Dynamic Code

At any time step the governing equation of the dynamic problem can be defined as eqn(2.8). Mass matrix [M] does not get affected by the plasticity. So for an incremental load the equation can be written in the elastoplastic region.

$$[M]^{e}\{\delta\ddot{U}\} + [K]^{ep}\{\delta U\} = \{\delta f\} \qquad (3.5a)$$

Again rewriting the above equation as we did for eqn(2.17), the equation acquires the following form,

$$[M]^{e}\{\delta\ddot{U}\} + [K]^{e}\{\delta U\} = \{\delta f\} + [[K]^{e} - [K]^{ep}]\{\delta U\} \qquad (3.5b)$$

Right hand side shows the sum of the original load and the unbalance. The unbalance arises because of the difference in elastic and plastic stress strain matrices. For every time step this equation is balanced to get an equilibrium displacement satisfying the plasticity in the element. Initial unbalance is taken to be zero and then a trial displacement vector is calculated. This process is continued until both side equalize.

The advantage of the eqn(3.5) is that the global stiffness matrix is not calculated repeatedly, and thus saving computer time.

3.4 Loading And Unloading

If a material becomes plastic while loading and then unloaded, it behaves purely elastically. To implement this aspect the history of the load is to be traced to keep track of the strains in the material. Once the material is unloaded the yield point also changes. So in case of plastic loading and unloading the yield point of every gauss point is different. In case of dynamic loading the arrival of plasticity may have two effects on the stability.

1. The displacements are higher than those in elastic case and thus functional aspect of the design gets affected.
2. The plasticity absorbs much energy, thus the impact of dynamic loading may reduce. Also in case of loading and unloading the energy which comes under hysteresis loop gets released, so the damping and stability in the system may increase.

3.5 Computer Implementation

The computer code developed by Dindore [1994] has been modified to take the elastoplastic nature of the material in the present work.

The present work deals with two type of loadings, uniform

distributed loading and moment loading on the boundary. Plastic code first determines the stresses and if these are still in the elastic region, the plastic part is bypassed. Also if ultimate stress is reached, the programme stops. If the load is in plastic region the load above the yield point is divided into 20 equal part and then applied in small steps. The core size of elasticity (h_p) and stresses are updated after each force iteration to save much computer time. The maximum error due to such scheme can be justified in following way.

Maximum stress generated < σ_{ult} < 1.5 σ_{yield}

$$so,\ stress\ increment < \frac{1.5\sigma_y - \sigma_y}{20} = 0.025\sigma_y$$

So the percentage of elastic height that will be missed due to above mentioned assumption in each force iteration is,

$$\delta h_p < \frac{0.025\sigma_y}{\sigma_y} x100 = 2.5\%$$

This error gets compensated to the extent because of following reasons,

1. Out of plane stresses are not varying with height.
2. Weightages of various gauss points average out the error.
3. Averaging out if the number of elements are more.

Out of seven gauss points any one or more may be elastic or plastic upto any height in the thickness. The results are shown as contour plots to show extent of plasticity on the plate

CHAPTER 4

RESULTS AND DISCUSSIONS

This Chapter is divided into two parts. Former consists of validation of computer code while later analyzes the interdependence of the plastic and dynamic behaviour.

4.1 Validation of code

Validation of computer code is done on following cases,

case 1. Stresses and deflections for the purely elastic problem are calculated for standard cases (Timeshenko,1959). A satisfactorily agreement (within 1.5 %) is observed with theoretical results. Mesh is used as in fig(4.2).

case 2. For qualitative validation, the plate is subjected to different loads and boundary conditions. The spread of plastic zone was observed and the same was more in the cases where either geometric curvature was more or there was a discontinuity in boundary conditions.

case 3. For quantitative validation, a standard problem is taken up (Hinton, 1984). In this a plate fixed in all edges, subjected to a uniform load is considered (fig 4.1). The material and geometric properties are as follows,

Modulus of elasticity, $E = 30000$ MPa

Poisson's ratio, $\nu = 0.30$

Density $\rho = 0.003 \times 10^6$ Kg/m^3

Thickness of plate, $h = 0.20$ m
Side of the square plate, $L = 6.0$ m
slope in plastic region, $K = 0.01 \times E$
Yield stress, $\sigma_y = 30.0$ MPa
Uniformly distributed load for $q = 0.30, 0.35, 0.40$ MN/m^2

The present finite element analysis consists of dividing the square plate by a uniform mesh as shown in fig(4.2). As can be seen quarter plate is also used by applying symmetric conditions at the boundaries.

Fig(4.3) presents the deflection at the centre of the plate as a function of load. The figure shows the results of considering the mesh as a full plate and as quarter plate(which is equivalent to a finer mesh with elements of half size) together with those semi-loof analysis [Hinton and Owen, 1980] and Hinton and Owen, 1984. It can be seen, that the present finite element results are in close agreement with semi-loof analysis. Fig(4.4-4.5) show the spread of plastic zone as contours plots at various load levels. At each load level the different contours correspond to percentage of plastic height. These plots also agree well with the results of Hinton (fig 4.4a) except toward peak load (q=0.40) Hinton's results behave more plastically. This happens because of layered approach used by Hinton. An inherent error of 12.5 percent is induced in dividing the thickness into 8 layers. And at peak loads the present results agree with those of Hinton's in this error range.

Thus, it can be seen that the results by the present code agrees well with the available quasi-static elastoplastic results.

4.2 Plastic and dynamic behaviour

This section presents elastoplastic dynamic results obtained by the present approach. The variation of dynamic load is assumed as follows,

$$q = \frac{q_0}{2}\left[1 - \cos\left(\frac{\pi\ t}{RT}\right)\right] \qquad t \le RT \qquad (4.1)$$

For load # 1,

$$q = q_0 \qquad t > RT$$

And for load # 2,

$$q = q_0 \cos\left(\frac{\pi\,(t - RT)}{2\ RT}\right) \qquad t > RT$$

Where RT is the rising time for the load to reach the peak. The forcing functions are shown in fig(4.6). The element size and time steps are choosen in such a way so that the solution remains stable.

To study the interdependence of plastic and dynamic behaviour, the analysis is divided into three parts.

I. The amount of plasticity which is the ratio of effective stress to yield stress in the most plastic gauss point, is shown against time for various rise time (fig 4.7). The load no. 1 is applied for this case. This is compared with the amount of plasticity in static case and also with onset of plasticity in

that element. For this case the amount of plasticity increases as the rise time is decreased while keeping other parameters same. By reducing the rise time the inertial forces play a significant role in inducing more plasticity.

II. For a given rise time the dynamic response is analyzed for purely elastic and elastoplastic cases. The load no. 2 is applied in this case. The deflection is also seen for different loads with same rising time. The deflection at the centre is plotted against time steps. Initial maximum defletion is more than the purely elastic case. But at latter time the delection is less (fig 4.8). For this case the dynamic response in elastoplastic case is more damped than the purely elastic case. The response becomes more dampened as the plasticity increases in the plate. The dissipation of energy in form of hysteresis losses accounts for this damping.

III. The number of gauss points which have gone plastic, are plotted against the time for different loads and given rise time. The load no. 2 is applied in this case (fig 4.9). For this case because of strain hardening the number of elastic gauss points is increasing during unloading. And at the next peak load the number of plastic gauss points is reducing than the previous one.

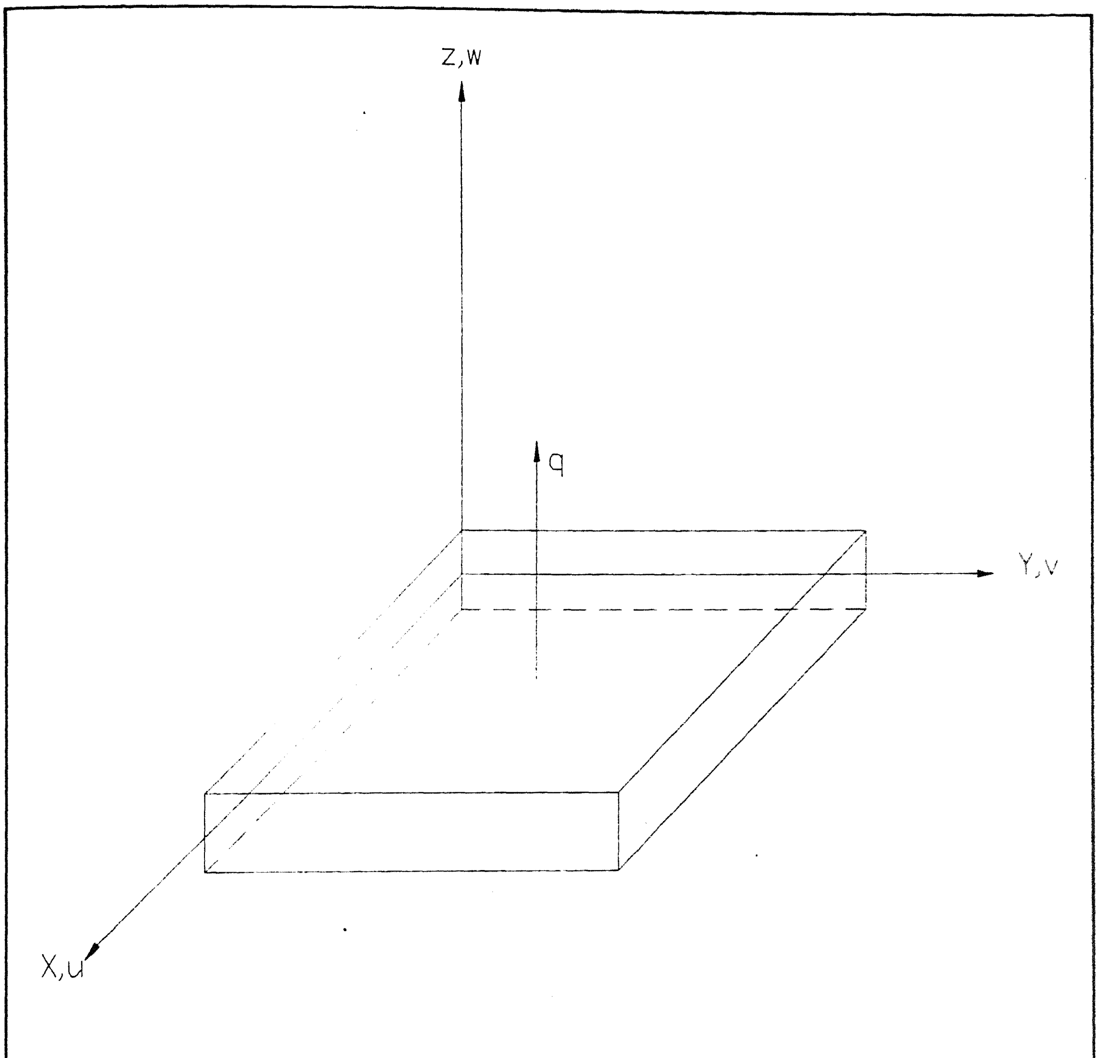
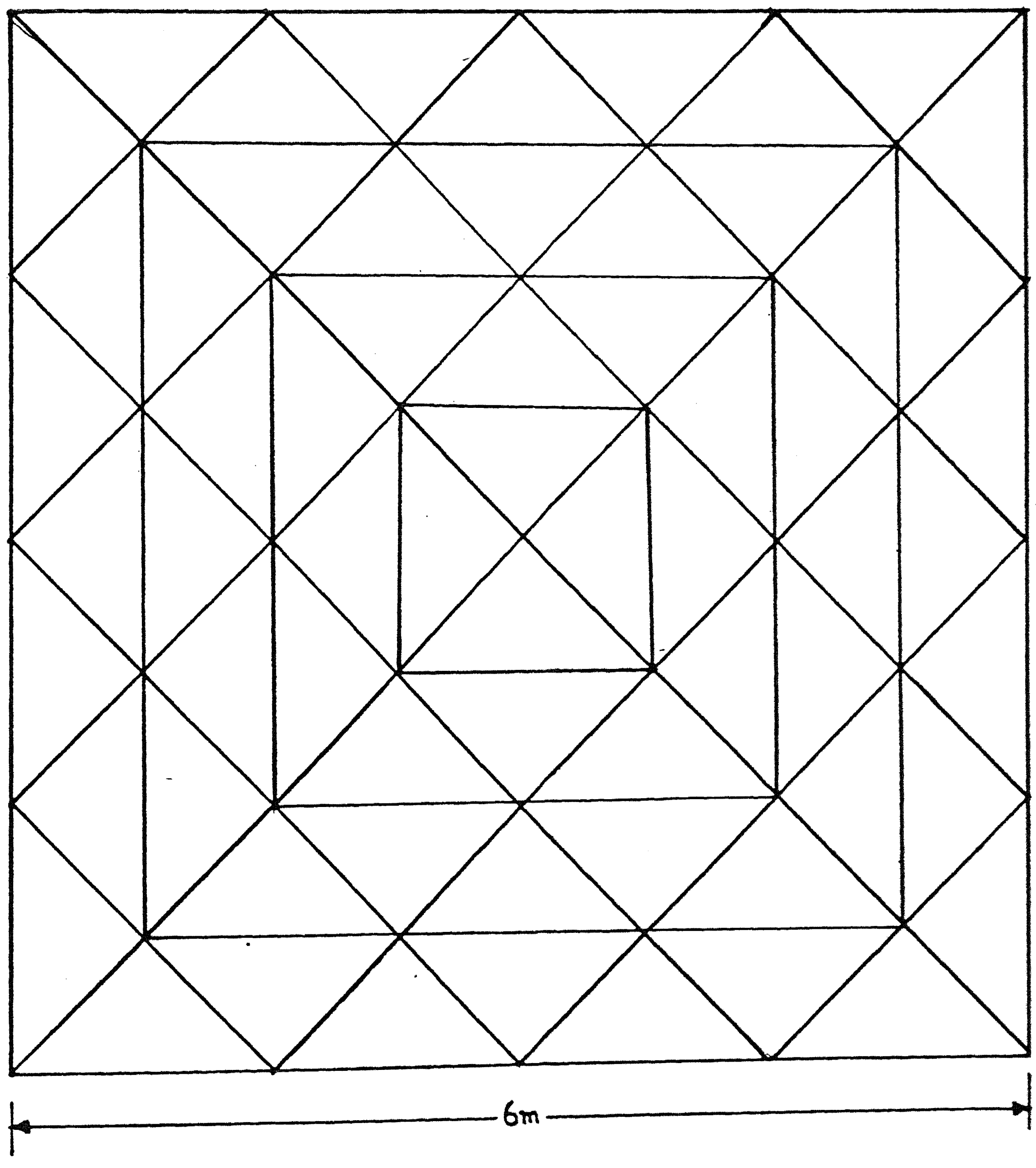


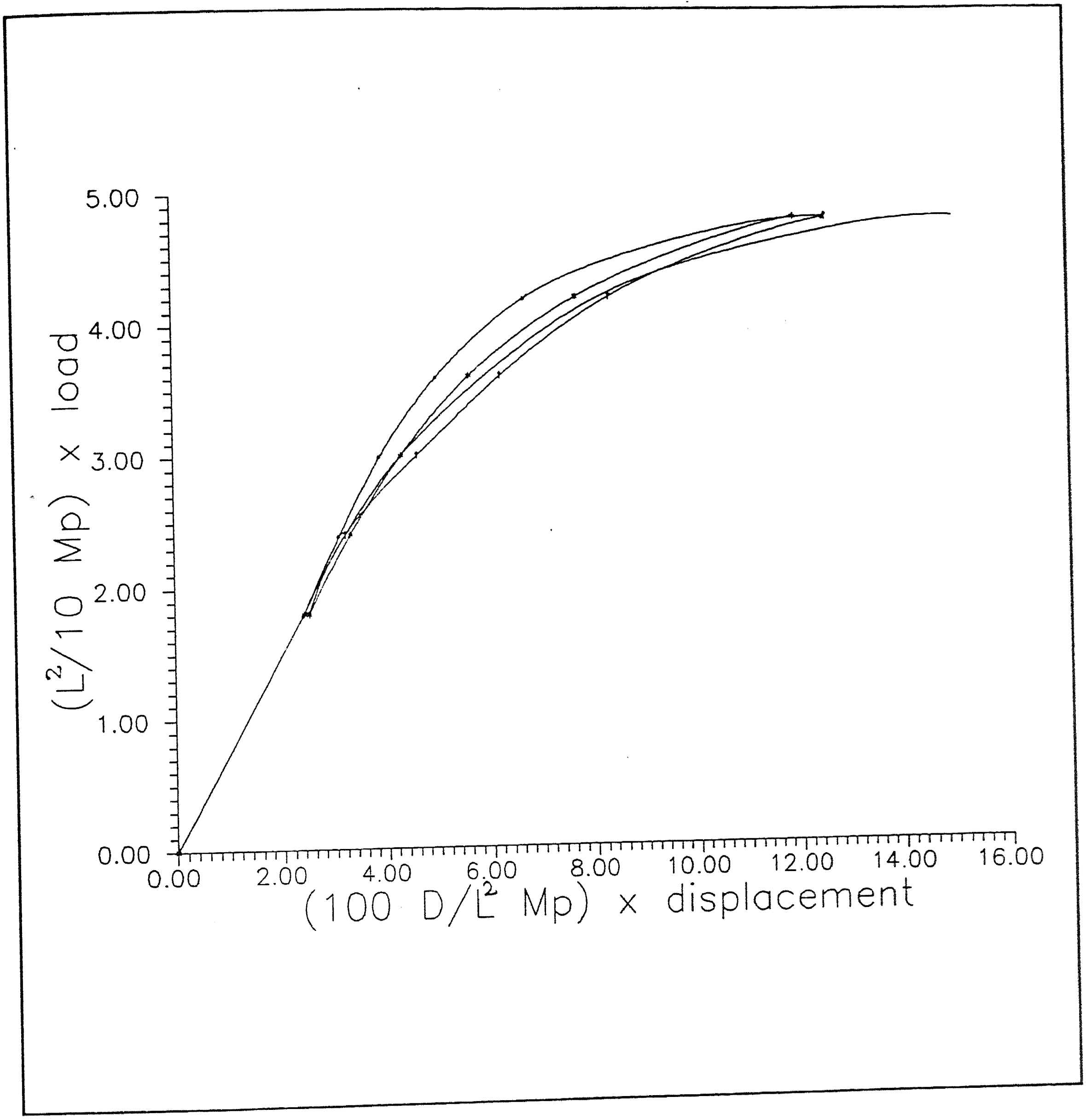
Fig (4.1)

Where q is the uniformly distributed load.

Fig(4.1)



Fig(4.2)



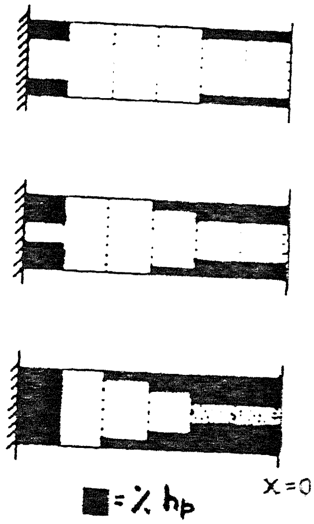
✓ - Semi loof analysis

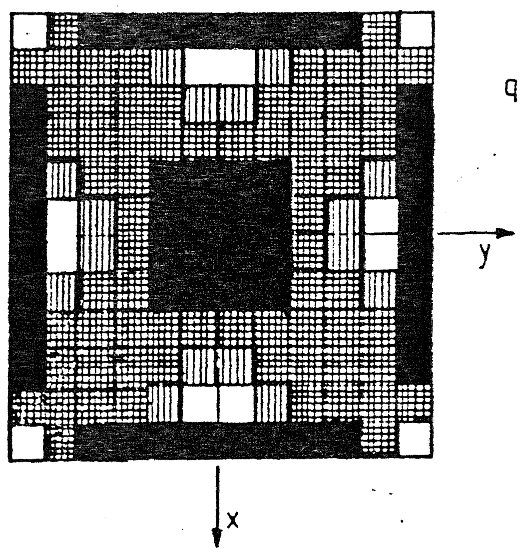
– - Hinton's analysis

↗ - Present analysis (full plate)

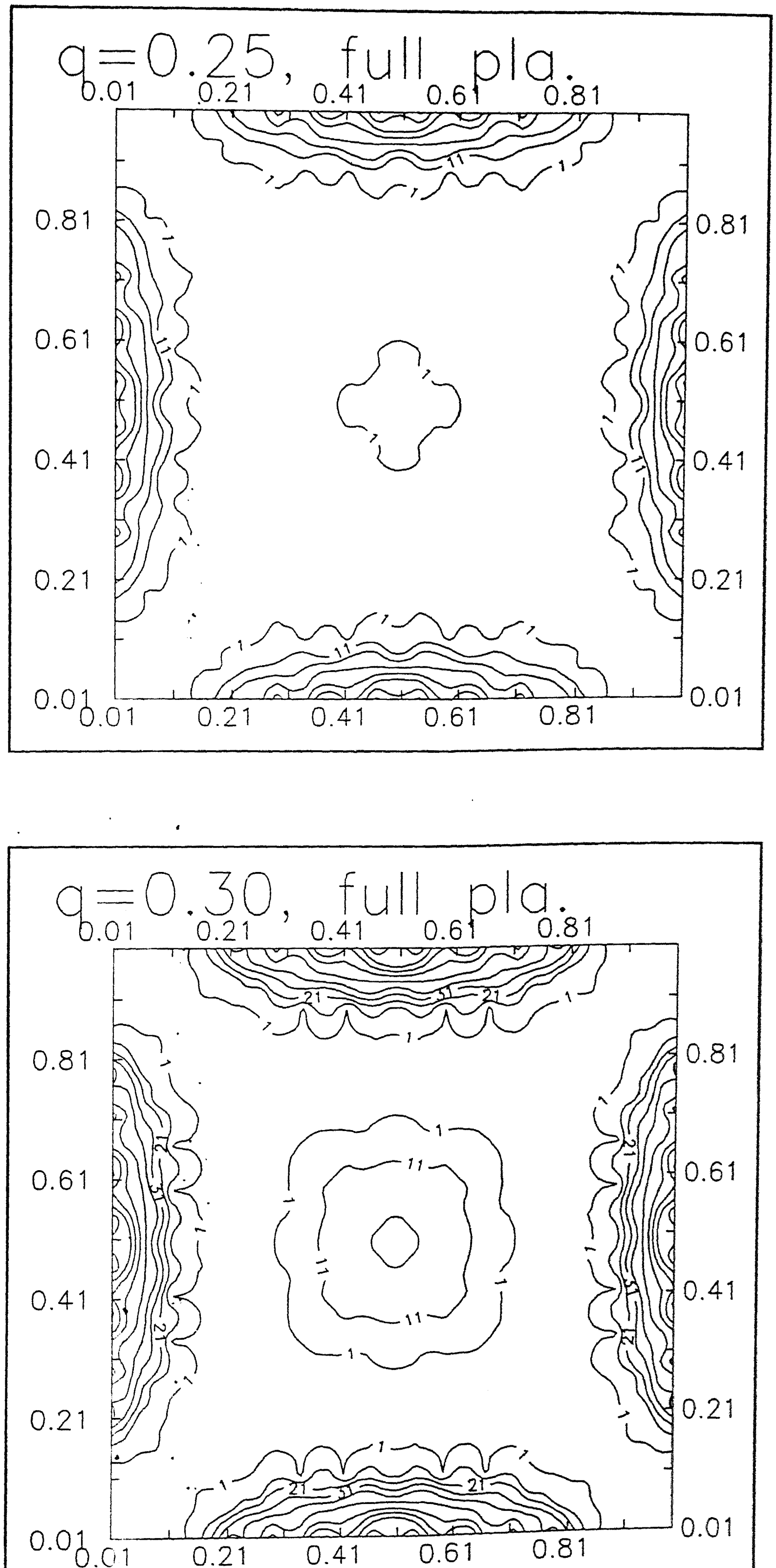
x - Present analysis (quarter plate)

Fig(4.3)

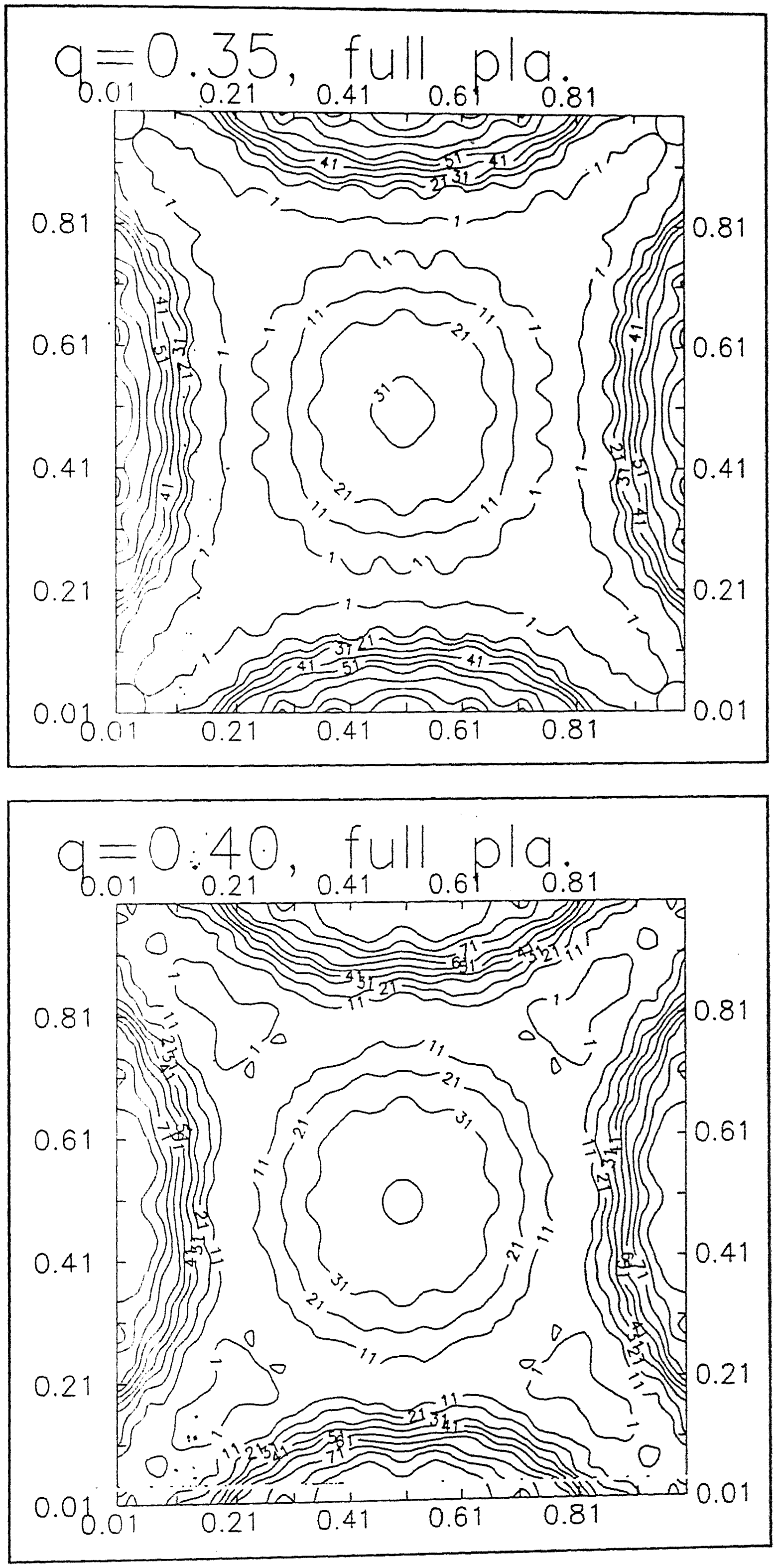




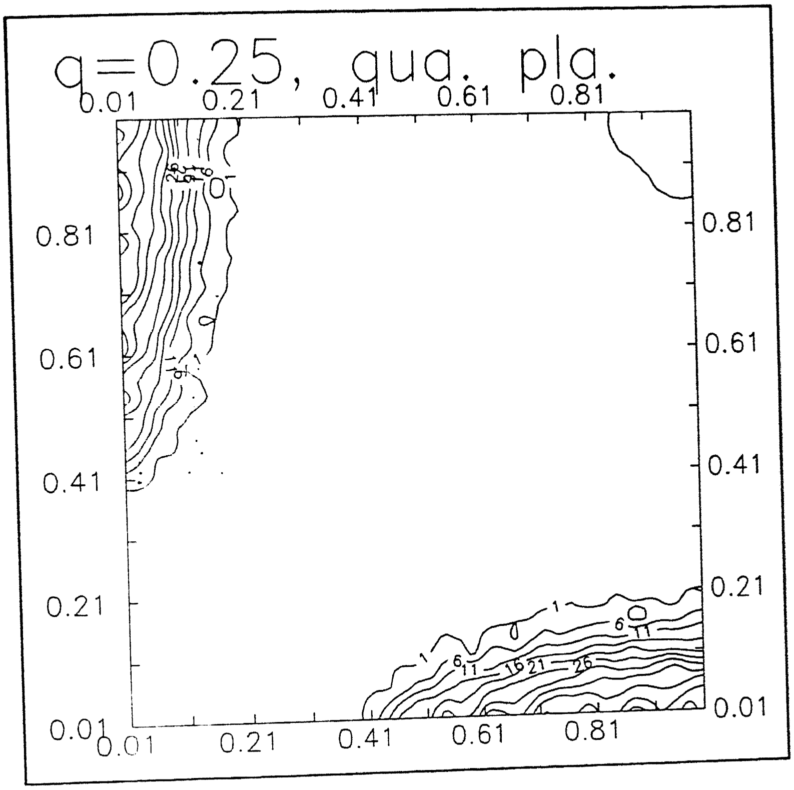
Fig(4.4a): standard results on height and spread of plasticity

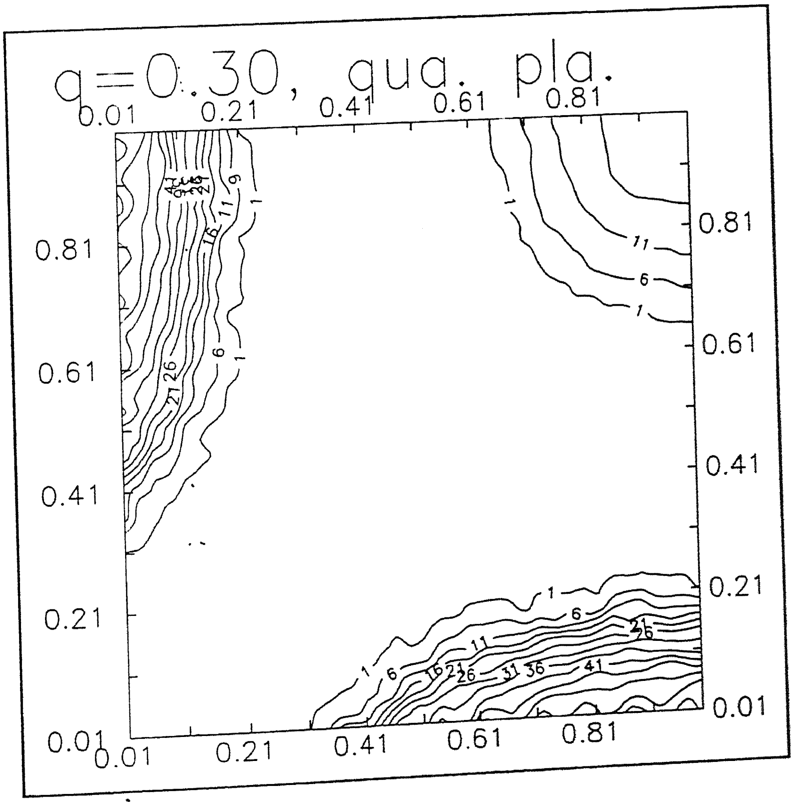


Fig(4.4b): contours show the percentage of plastic height

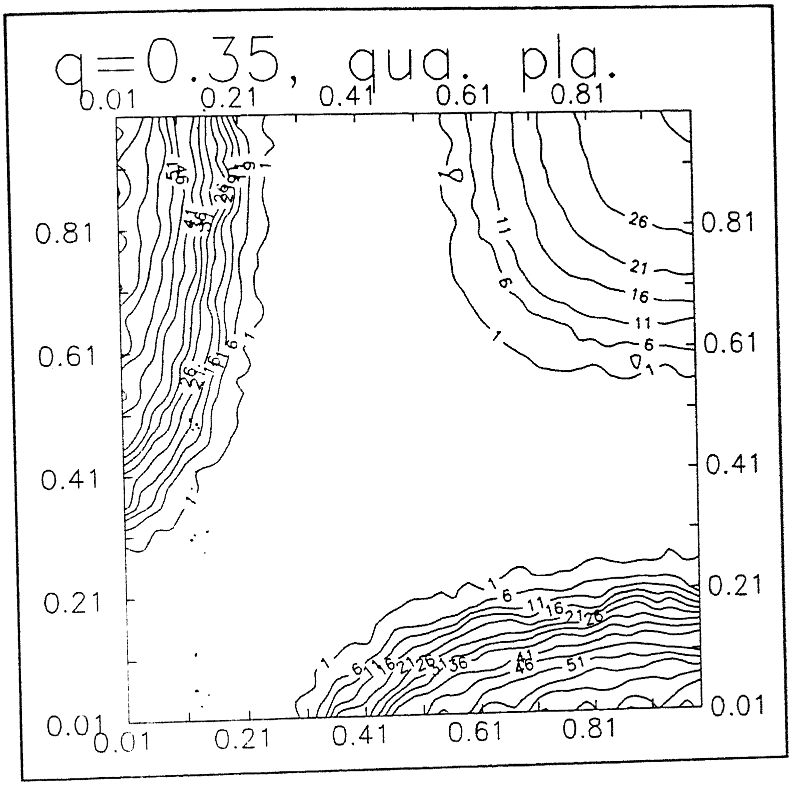


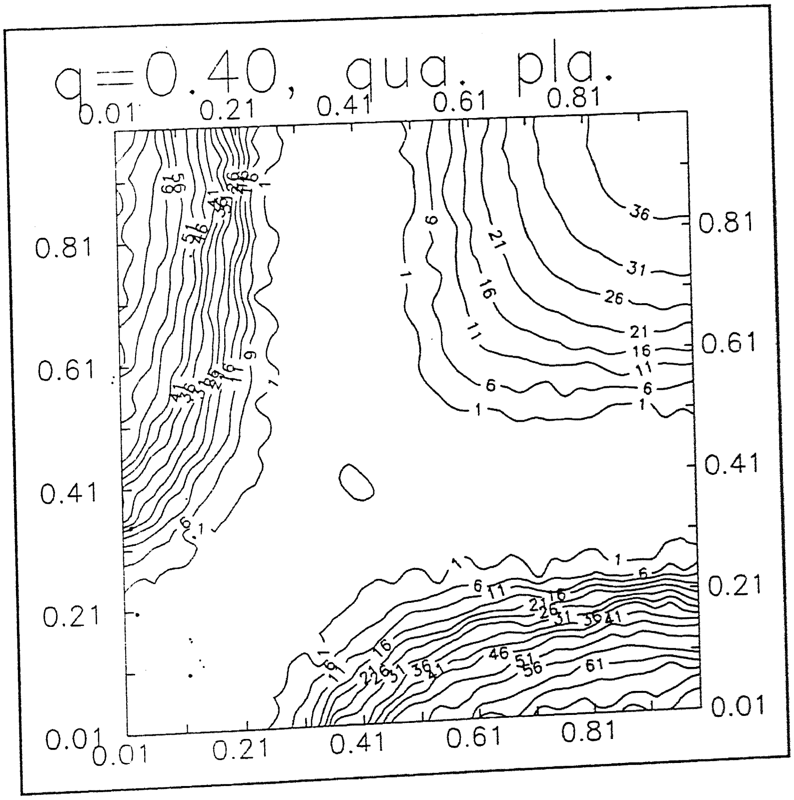
Fig(4.4c): Contours show the percentage of plastic height.



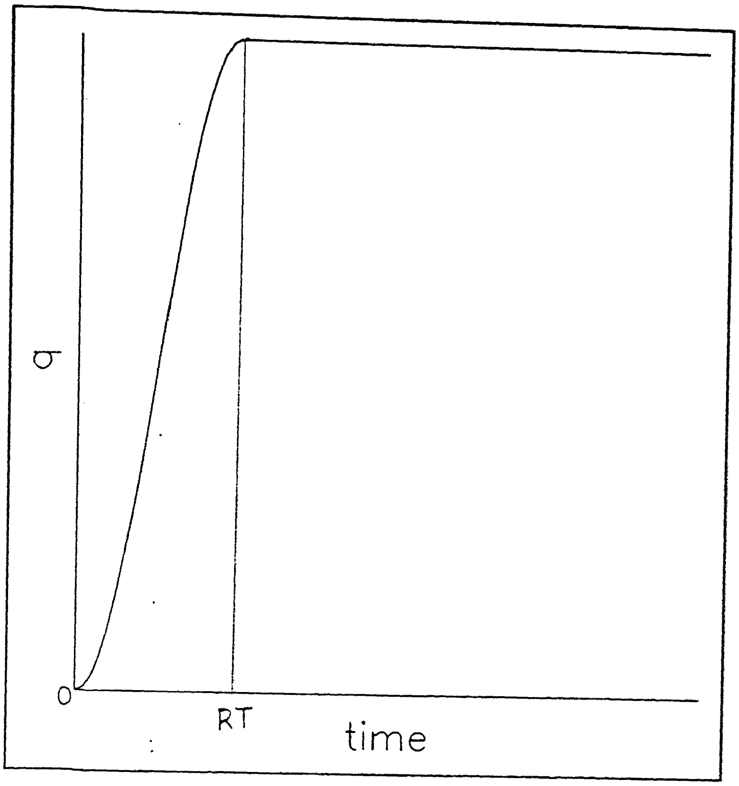


Fig(4.5a): Contours show the percentage of height of plasticity

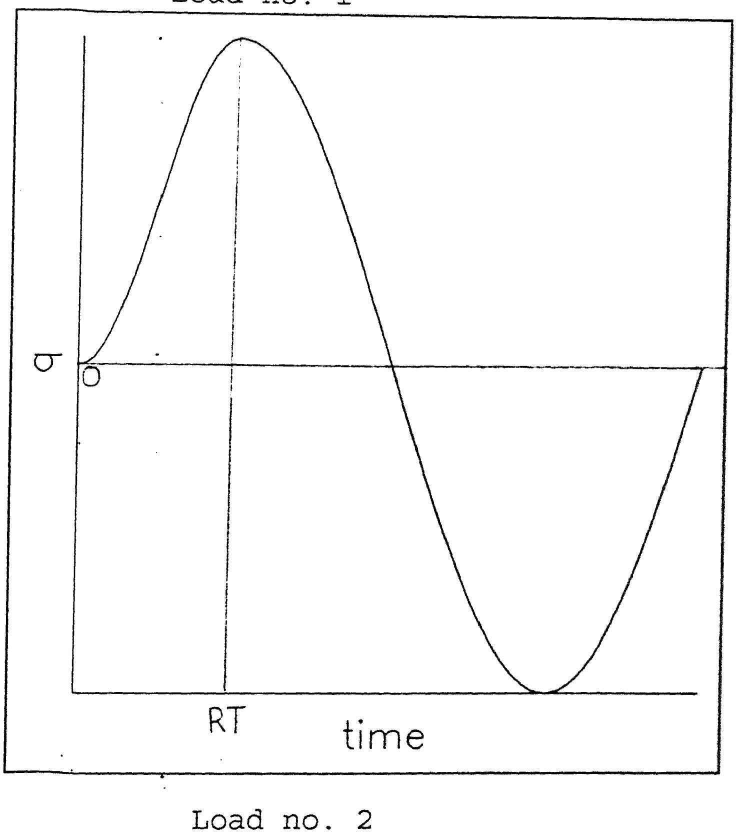




Fig(4.5b): Contours show the percentage of height of plasticity

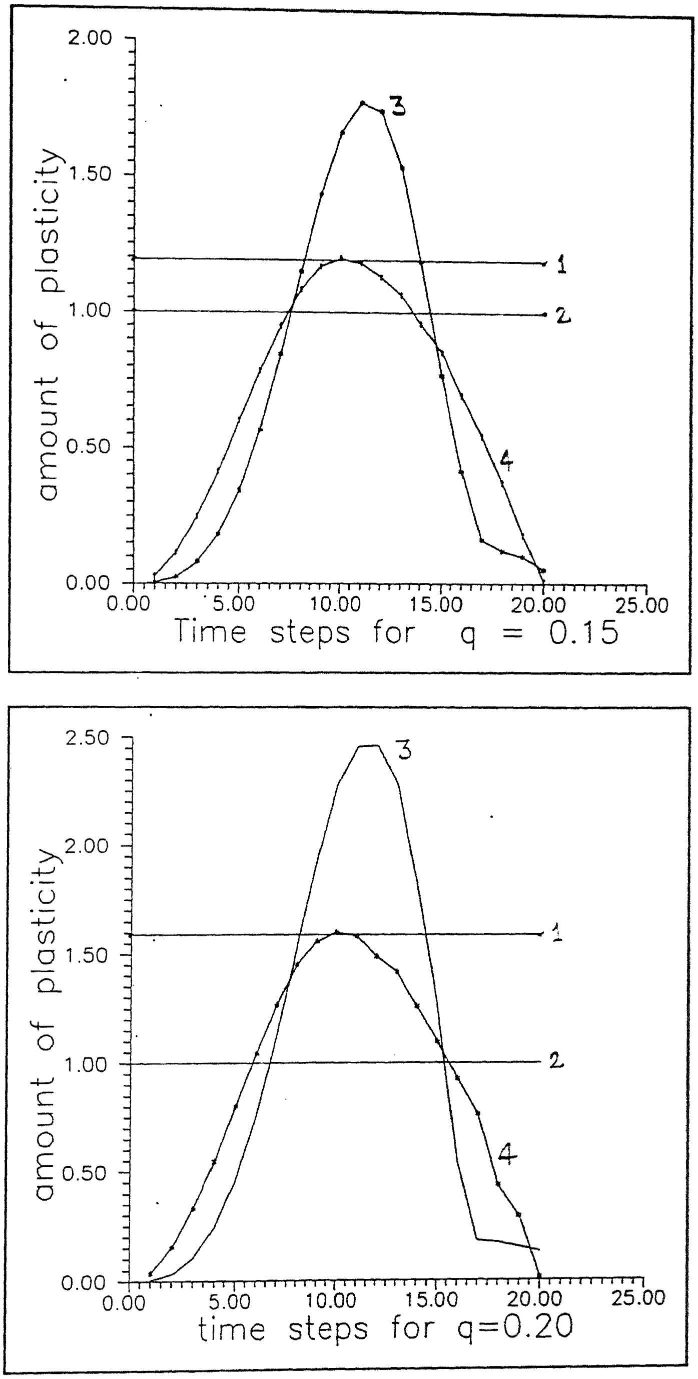


Load no. 1



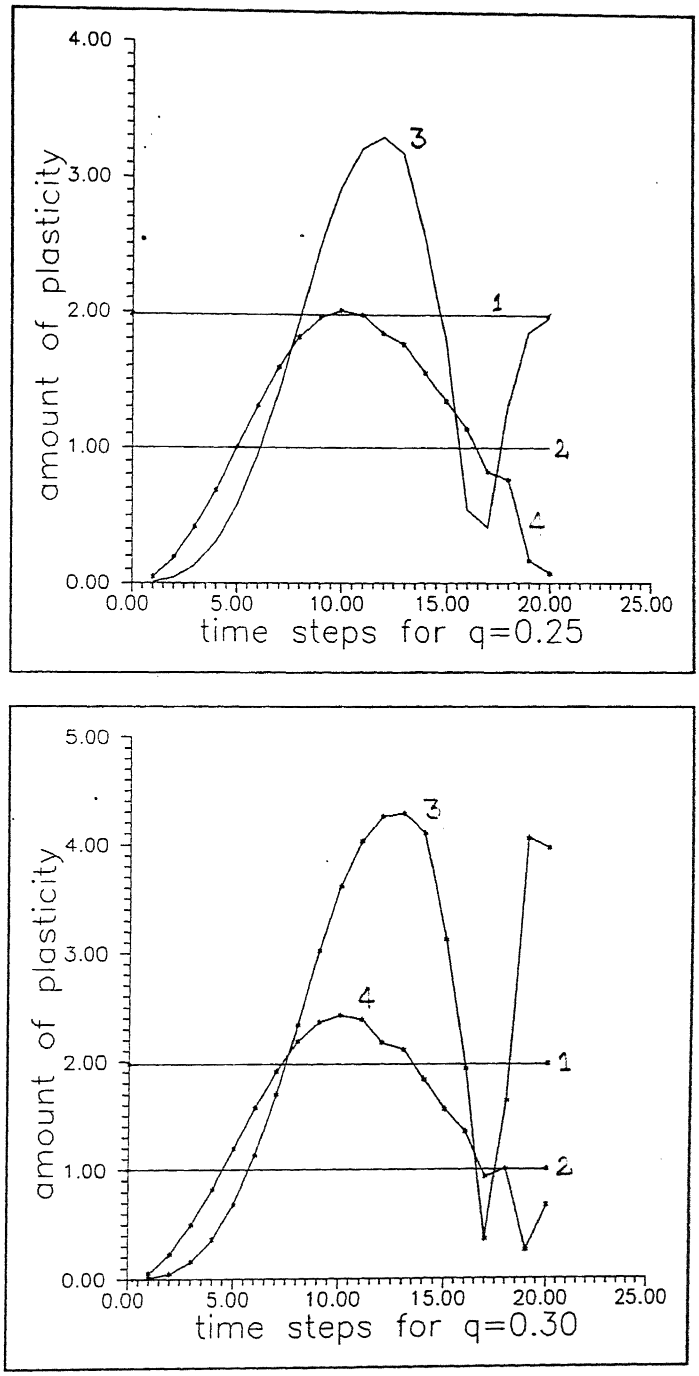
Load no. 2

Fig(4.6): Shows different type of forcing functions.



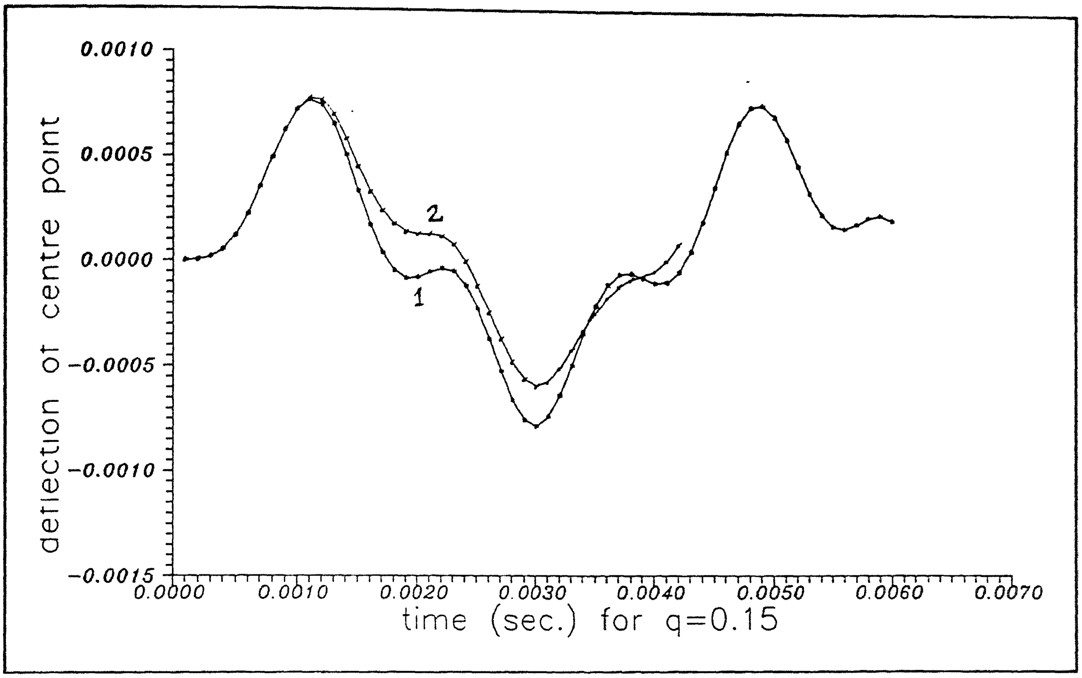
1 - Static case
2 - Onset of plasticity
3 - For rise time = 0.001
4 - For rise time = 0.01

Fig(4.7a): Effect of dynamic loading on the plasticity.

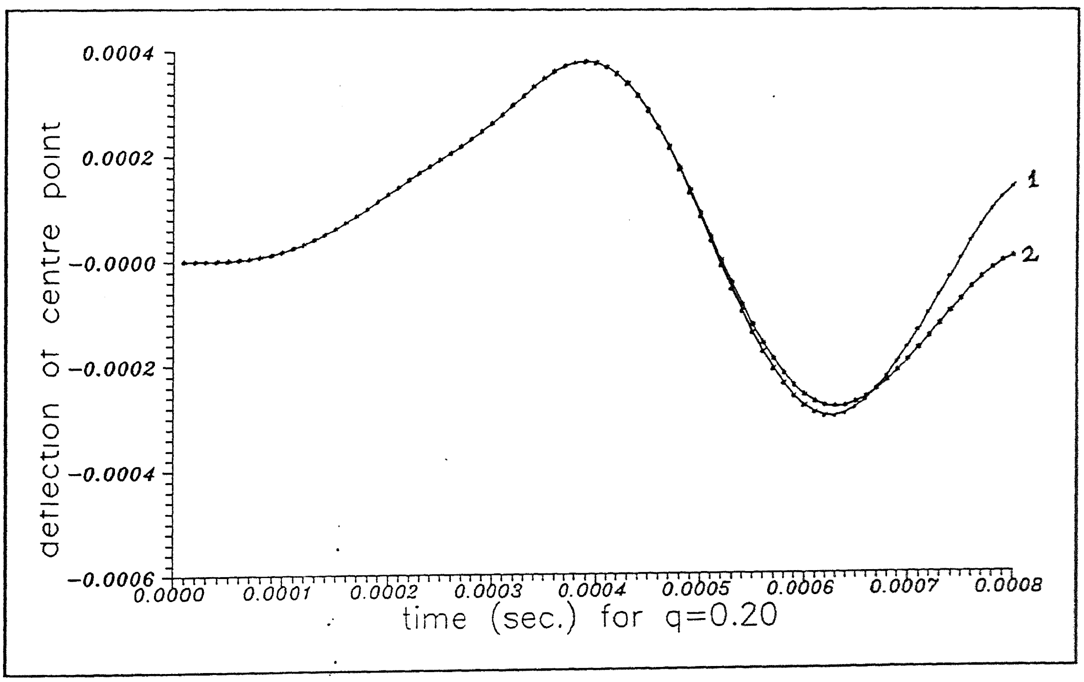


1- Static case
2- Onset of plasticity
3- For rise time =0.001
4- For rise time =0.01

Fig(4.7b): Effect of dynamic loading on plasticity.



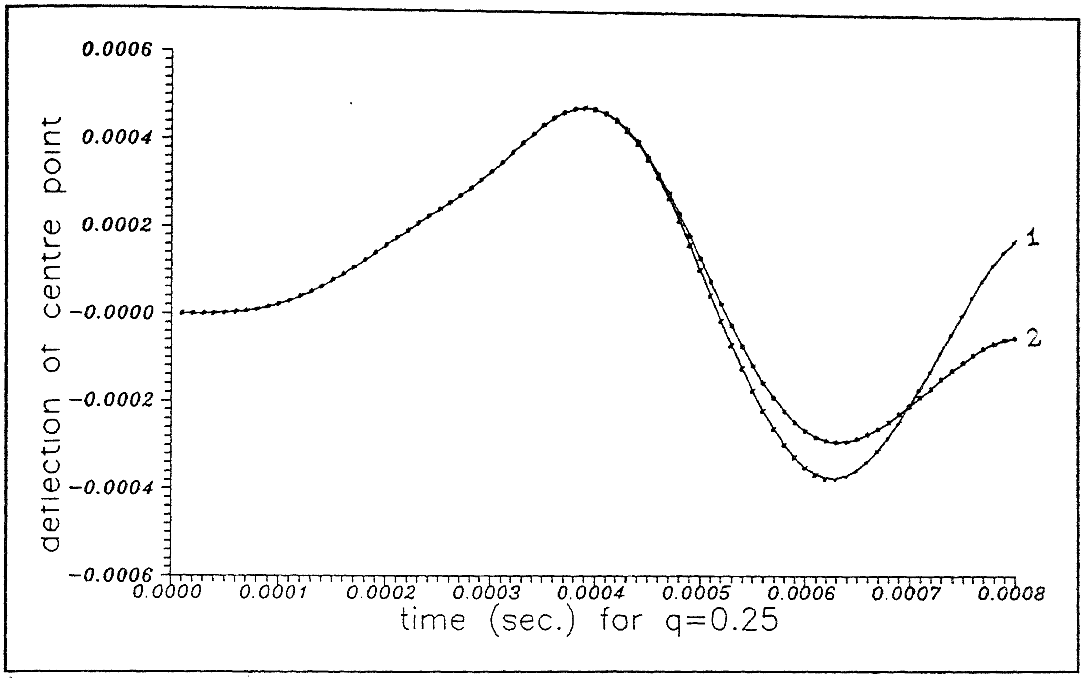
RT=0.001



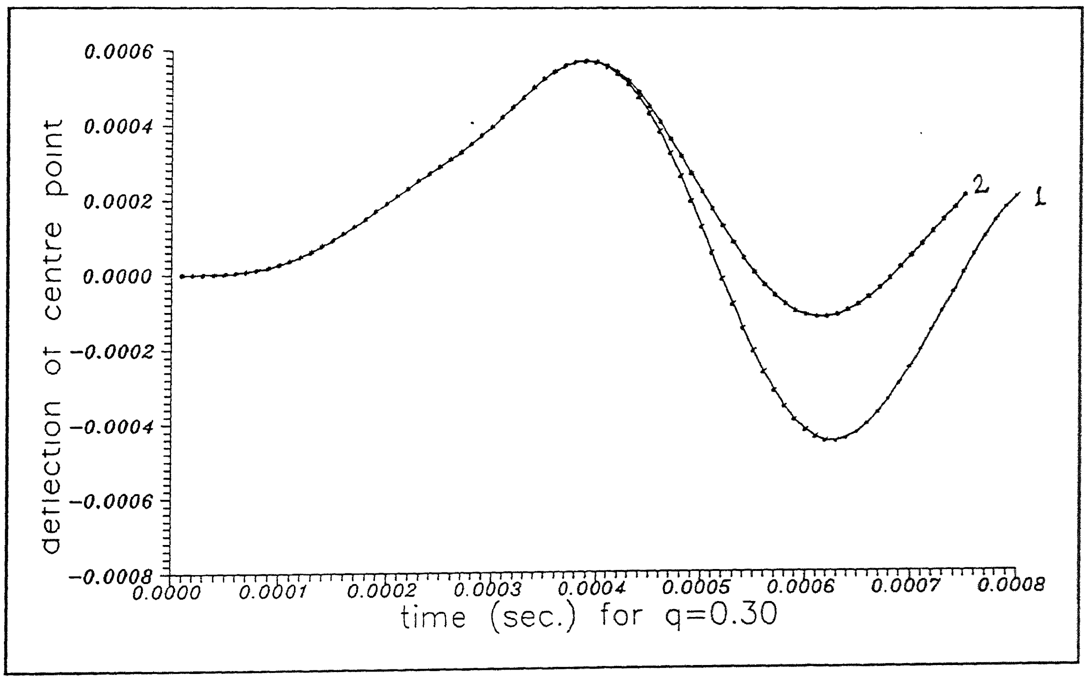
RT=0.0001

1- Purely elastic response
2- Elastoplastic response

Fig(4.8a): Effect of plastic behaviour on dynamic response



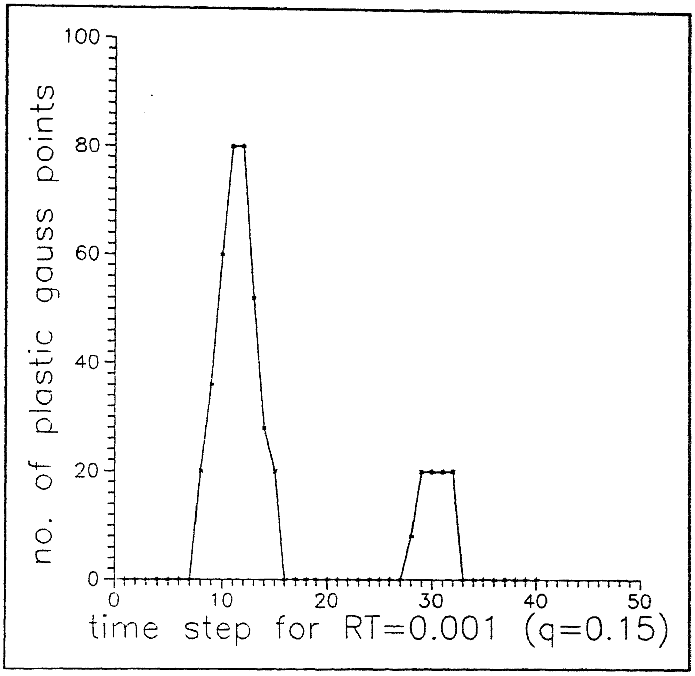
RT=0.0001

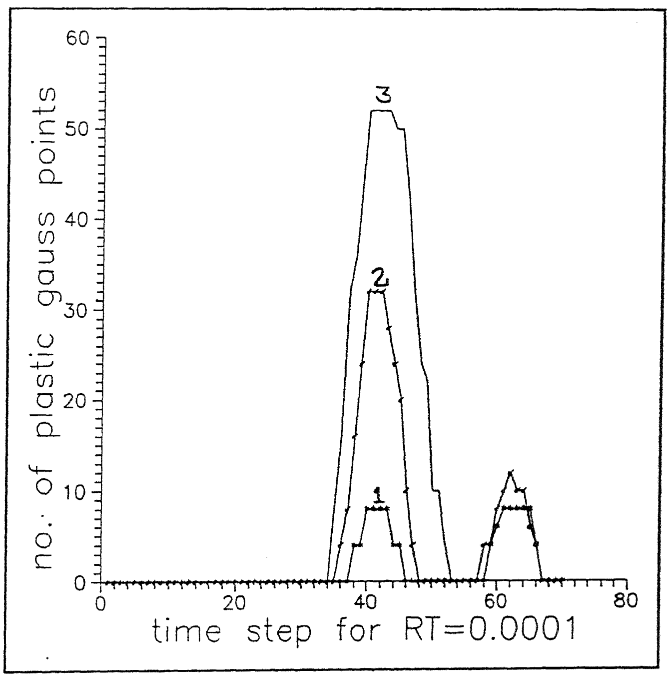


RT=0.0001

1- Purely elastic case
2- Elastoplastic case

Fig(4.8b): effect of plastic behaviour on dynamic response





1- q=0.20
2- q=0.25
3- q=0.30

Fig(4.9): Effect of strain hardening in loading and unloading. Peaks of the curves correspond to that of forcing function.

CHAPTER 5

CONCLUSIONS AND SCOPE FOR FUTURE WORK

5.1 Conclusions

Based on the present finite elastoplastic and dynamic analysis of a plate under dynamic and static loading, following conclusions can be verified,

1. In case of dynamic loading on the plate the plasticity is more than it was in purely static load with the same peak value.

2. The plasticity keeps on increasing as the rise time is decreased. And for a sufficient longer rise time there is no additional increase in the amount of plasticity over the static case.

3. As the plasticity is induced in the system, the energy is dissipated which acts as a dampener on the dynamic response and thus the oscillations reduces or vanishes.

4. The deflection during first peak load is more in elastoplastic case than purely elastic case. And in latter load peaks because of damping the deflection comes down for elstoplastic case.

5.2 Future Scope Of Work

The mesh can be made finer and the time step can be made smaller to observe the sharper impact loading and plasticity more accurately.

Several other loadings and different geometrical configurations can be tried. Also the cases of cyclic loading can be studied to see the permanent deflection after a given number of cycles.

Normally the small cracks are always present in the material, but that can not be modelled in the finite element. So to model the cracks we can make use of statistical method with varying yield stresses at different gauss points.

The effect of plasticity acts as an dampener in the dynamic response. The study can be useful where the system vibrations are to be avoided at the cost of permanent damage of some parts of the system.

For dynamic loading the time step can be selected in a optimum manner so that the computational time is less with considerable accuracy and stability.

REFERENCES

Ante, A. , 1987,'The explosive welding of metals.',First Int. Con. on Computational Plasticity, Barcelona.

Bathe, K. J. , 1990, Finite Element Procedures in Engineering Analysis, Prentice Hall of India Pvt. Ltd.

Borino, G., Caddemi, S. and Pollizzato, C., 1987,'Mathematical programming methods for evaluating dynamic deformations of elastoplastic structures.',First Int. Con. on Computational Plasticity, Barcelona.

Bazant, Z. P. , 1978, 'Spurios reflections of elastic waves in non uniform finite element grids. ', Comp. Meth. Appl. Mech. Engg, 16, 91-100.

Bazant, Z. P., Feng-Bao Lin and Pijaudier C. J., 1987,'Yield limit degradation: non local continuum model with local strain.', First Int. Con. on Computational Plasticity, Barcelona.

Bazant, Z. P. and Celep, Z., 1983, 'Spurious reflections of elastic waves due to gradually changing finite element size.' Int. J. Num. Meth. Engg., 19, 631-646.

Casadei, F., Halleux, J. P., Verzeletti, G. and Youstsos, 1987,' Application of numerical models to dynamic flow of steel specimens in the large test facility.', First Int. Con. on Computational Plasticity, Barcelona.

Dindore, U. S. , 1994, 'Dynamic effect of impact load on a plate with a crack: A finite element analysis', M. Tech. Thesis, I.I.T. Kanpur.

Giulio Maier and Giorgio Novati, 1987,'Deformation bounds for elastoplastic discrete structures with piecewise linear yield locus and non-linear hardening.',First Int. Con. on Computational Plasticity, Barcelona.

Hinton, E. and Owen, D.R.J., 1984, Software For Finite Element Methods in Plates and Shells., Pineridge Press, Swansea.

Hinton, E. and Owen, D.R.J., 1980, Finite Element in Plasticity: Theory and Practices., Pineridge Press, Swansea.

Kujawski, J., Miedzialowski and C., Ryzynski, W., 1987,'Dynamic analysis elastoplastic reinforced concrete wall systems.' First Int. Con. on Computational Plasticity, Barcelona.

Marques, J. M. M. C., 1987,'Code validation case studies in non-linear dynamic finite element analysis.',First Int. Con. on Computational Plasticity, Barcelona.

Mazza, G., Minasola, R., Molinaro, P. and Papa, A., 1987,'Dynamic

structural response of steel slab doors to blast loading.',First Int. Con. on Computational Plasticity, Barcelona.

Nayak, G. C., 1971,'Plasticity and large deformation problems by finite element method.', Ph. D. thesis, University of Wales, Swansea.

Nayak, G. C. and Zienkiewicz, O. C., 1972,'Elastoplastic stress analysis: Generalization for constitutive relations including strain softening.', Int. J. Num. Meth. Engg., 5, 113-35.

Owen, D. R. J. and Li, Z. H., 1987, 'Elastoplastic numerical analysis of anisotropic laminated plates by a refined finite element model.', First Int. Con. on Computational Plasticity, Barcelona.

Panzeca, T. and Polizzatto, C., 1987,'A finite element model for dynamic elastoplastic structural analysis.',First Int. Con. on Computational Plasticity, Barcelona.

Seron, F. J., 1990, , Finite element method for elastic wave propagation', Commun. In Appl. Num. Meth. ,6 ,359-368

Simo, J. C. and Taylor, R. L., 1985, 'Consistent tangent operators for rate dependent elastoplasticity.', Com. Meth. Appl. Mech. Engg., 48, 101-18

Timeshenko, S. and Woinowsky-Krieger, S., 1959, Theory of plates and shells, McGraw Hill Book Company Inc.

Yamada, Y., Yishimura, N. and Sakurai, T, 1968, 'Plastic stress strain matrix and its application for the solution of elastoplastic problems by finite element method.', Int. J. Mech. Sci., 10, 343-54.

Zienkiewicz, O. C., Qu, S., Taylor, R. L. and Nakazawa, S. , 1986, 'The patch test for mixed formulation.', Int. J. Numer. Meth., 23,1873-1887

Zienkiewicz, O. C. and Lefebvre, D.,1988, 'A robust triangular plate bending element of the Reissner-Mindlin type.', Int. J. Num. Meth., 26, 1167-1184

Zienkiewicz, O. C. and Taylor, R. L., 1991, The Finite Element Method, Fourth edition , vol. II, McGraw Hill Book Company.